34/35

Differential Calculus

Lab Assignment 4

1. Find an explicit formula for the inverse of the function. Also find the domain and range of f and f^{-1}.

   (a) (4 marks) $f(x) = \sqrt{3+9x}$.

   (b) (4 marks) $f(x) = e^{x-4}$.

   (c) (6 marks) $f(x) = \dfrac{8x+1}{5x-3}$.

2. Solve each equation/inequality for x.

   (a) (5 marks) $\ln x + \ln(x-2) = \ln 35$.

   (b) (4 marks) $e^{3-5x} > 7$.

3. Evaluate the following without using a calculator (Show your steps):

   (a) (5 marks) $\sin\left(\cos^{-1}\left(\dfrac{12}{13}\right)\right)$ $= 5/13$

   (b) (5 marks) $\cos\left(\tan^{-1}\left(\dfrac{-3}{5}\right)\right)$ $5/\sqrt{34}$

   (c) (2 marks) $\tan^{-1}\left(\tan\left(\dfrac{-\pi}{7}\right)\right)$

$f(x)=\sqrt{3+9x}$

$\sqrt{3+9x} \geq 0$

$x \geq -\frac{3}{9} = -\frac{1}{3}$

1)a) $y=\sqrt{3+9x}$

$x=\sqrt{3+9y}$

$f(x)\begin{cases}\text{Domain} \rightarrow D=\{x\in\mathbb{R} \mid x\geq -1/3\} \\ \text{Range} \rightarrow R=\{y\in\mathbb{R} \mid y\geq 0\}\end{cases}$ ✓

4 $y=\frac{x^2-3}{9}$ → Inverse → $f^{-1}(x)=\frac{x^2-3}{9}$

$f^{-1}(x)\begin{cases}\text{Domain} \rightarrow D=\{x\in\mathbb{R} \mid x\geq 0\} \\ \text{Range} \rightarrow R=\{y\in\mathbb{R} \mid y\geq -1/3\}\end{cases}$

1b) $f(x)=e^{x-4}$

$y=e^{x-4}$

$x=e^{y-4}$

$\ln x = \ln e^{y-4}$

$\ln x = y-4$

$y=\ln x+4$

4

$f(x)\begin{cases}\text{Domain} \rightarrow \{x\in\mathbb{R}\} \\ \text{Range} \rightarrow \{y\in\mathbb{R} \mid y\geq 0\}\end{cases}$

$f^{-1}(x)=\begin{cases}\text{Domain} \rightarrow \{x\in\mathbb{R} \mid x\geq 0\} \\ \text{Range} \rightarrow \{y\in\mathbb{R}\}\end{cases}$

1c) $f(x)=\frac{8x+1}{5x-3}$

$f(x)\begin{cases}\text{Domain} \rightarrow D=\{x\in\mathbb{R} \mid x\neq 3/5\} \\ \text{Range} \rightarrow R=\{y\in\mathbb{R} \mid y\neq 8/5\}\end{cases}$ ✓

$y=\frac{8x+1}{5x-3}$

$5xy-3y=8x+1$

$5xy-8x=1+3y$

$x(5y-8)=1+3y$

$x=\frac{1+3y}{5y-8}$

6

$f^{-1}(x)\begin{cases}\text{Domain} \rightarrow D=\{x\in\mathbb{R} \mid x\neq 8/5\} \\ \text{Range} \rightarrow R=\{y\in\mathbb{R} \mid y\neq 3/5\}\end{cases}$

$f^{-1}(x)=y=\frac{1+3x}{5y-8}$

4

2a) $\ln x + \ln(x-2) = \ln 35$ ✓

$\ln[(x)(x-2)] = \ln 35$ ✓

$\ln(x^2-2x) = \ln 35$ ✓

$x^2 - 2x = 35$ ✓

$x^2 - 2x - 35 = 0$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{2 \pm \sqrt{4-4(-35)}}{2}$ ✓

7

-5 ✗

check

Sub in 7 → $\ln 7 + \ln(5) = \ln 35$ → True

Sub in -5 → $\ln(-5) + \ln(-7) = \ln 35$ → False ✓

∴ the solution is $x = 7$

4

2b) $e^{3-5x} > 7$ ✓

$(3-5x)\ln e > \ln 7$ ✓

$3 - 5x > \ln 7$

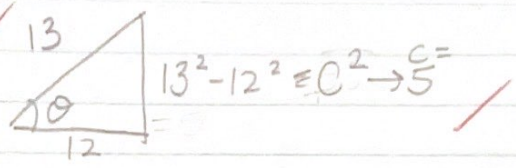
$x < \frac{\ln 7 - 3}{-5} \rightarrow x < \frac{3 - \ln 7}{5}$ ✓

3a) $\sin(\cos^{-1}(12/13))$

$\sin(\sin^{-1}(5/13))$

$= 5/13$

5

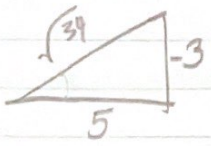


3b) $\cos(\tan^{-1}(-3/5))$

$\cos(\cos^{-1}(5/\sqrt{34}))$

$= \frac{5}{\sqrt{34}}$

5



3c) $\tan^{-1}(\tan(-\pi/7)) = -\frac{\pi}{7}$

$f^{-1}(f(x)) = x$

2