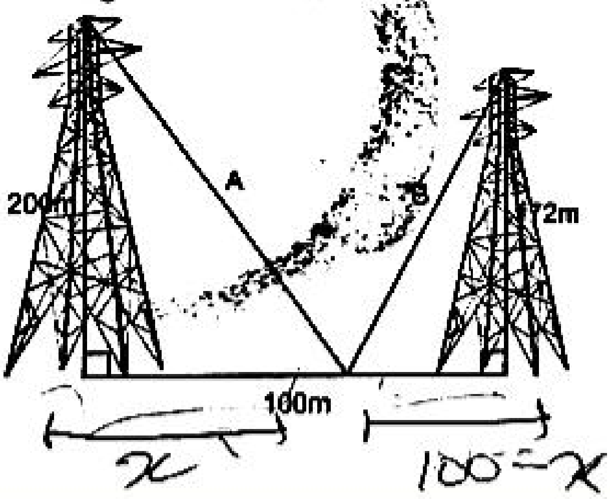
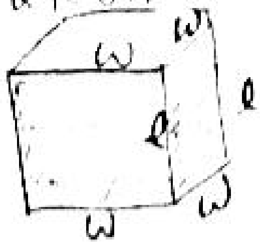
1. (10 marks) If 2700 cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box.

2. (10 marks) A piece of wire 12m long is cut into two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is (a) a maximum (b) a minimum

3. (10 marks) Two radio towers, of height 172m and 200m, are placed 100m apart. We wish to secure the two towers with a single cable connecting the top of each tower to a point on the ground between them. Where should the cable be placed to minimize the length of the cable?



1) $A = 2700\,cm^2$



Surface Area: $4lw + w^2$

$$2700 = 4lw + w^2$$

$$\frac{2700 - w^2}{4w} = l$$

$$V = lwh = lw^2$$

$$V = \left(\frac{2700 - w^2}{4w}\right) \cdot w^2$$

$$V = \frac{2700w^2 - w^4}{4w}$$

$$V = \frac{\cancel{w^2}^{\,w}(2700 - w^2)}{4\cancel{w}}$$

$$V = \frac{w(2700 - w^2)}{4}$$

$$V(w) = \frac{2700w - w^3}{4}$$

$$V'(w) = \frac{1}{4}\left(2700w - w^3\right)'$$

$$V'(w) = \frac{1}{4}\left(2700 - 3w^2\right)$$

Let the length, width, volume, and area be l, w, V, and A, respectively

$$0 < l < \infty$$

$$0 < w < \infty$$

$V'(w) = 0$	$V'(w)$ DNE
$V'(w) = 675 - \frac{3}{4}w^2$ $0 = 675 - \frac{3}{4}w^2$ $-675 = -\frac{3}{4}w^2$ $w = 30$	exists everywhere it's a polynomial

No endpoints → 2nd derivative test

$$V''(w) = \left(675 - \frac{3}{4}w^2\right)'$$

$$= -\frac{3}{4} \cdot 2w$$

$$= -\frac{6}{4}w$$

$$= -\frac{3}{2}w$$

↳ a max

Length when $w = 30\,cm$

$$\frac{2700 - w^2}{4w} = \frac{2700 - 30^2}{4(30)} = 15\,cm.$$

plug in

$$V = l^2h$$

$$= 13500\,cm^3$$

∴ Width should be 30cm and height should be 15cm in order to maximize Volume.

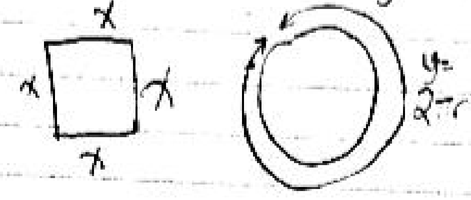
Hilary

2 12m

maximize?

Let r be radius of circle cm

Let the length of each side of the square be x, and the length of the circle be y cm. Let the total area be A m^2.



$4x + 2\pi r = 12m \rightarrow x = \frac{12-2\pi r}{4}$

$A = \pi r^2 + x^2$

area of circle area of square

$A(r) = \pi r^2 + \left(\frac{12-2\pi r}{4}\right)^2$

$A'(r) = \pi(2r) + 2\left(\frac{12-2\pi r}{4}\right) \cdot \frac{1}{4}(12-2\pi r)'$

$A'(r) = 2\pi r + \frac{1}{2}\left(\frac{12-2\pi r}{4}\right)(2\pi)$.

$A'(r)=0$	$A'(r)$ DNE
$0 = 2\pi r + \pi\left(\frac{12-2\pi r}{4}\right)$	exists everywhere
$0 = \frac{8\pi r + 12\pi - 2\pi^2 r}{4}$	
$0 = 2\pi(4r + 6 - \pi r)$	
$0 = 4r + 6 - \pi r$	
$-6 = 4r - \pi r$	
$-6 = r(4-\pi)$	
$\frac{-6}{4-\pi} = r$	
$r \approx -6.9896$	
doesn't count, not in interval	

Constraints

If $2\pi r = 0$

$4x + 2\pi r = 12$

$4x + 0 = 12$

$4x = 12$

$x = 3$

$0 \leq x \leq 3$

If $x = 0$ $\boxed{2\pi r = y}$

$4(0) + 2\pi r = 12$

$2\pi r = 12$

$(y = 12)$

$0 \leq y \leq 12$

circumference

$2\pi r = 12 \rightarrow r = 6/\pi$

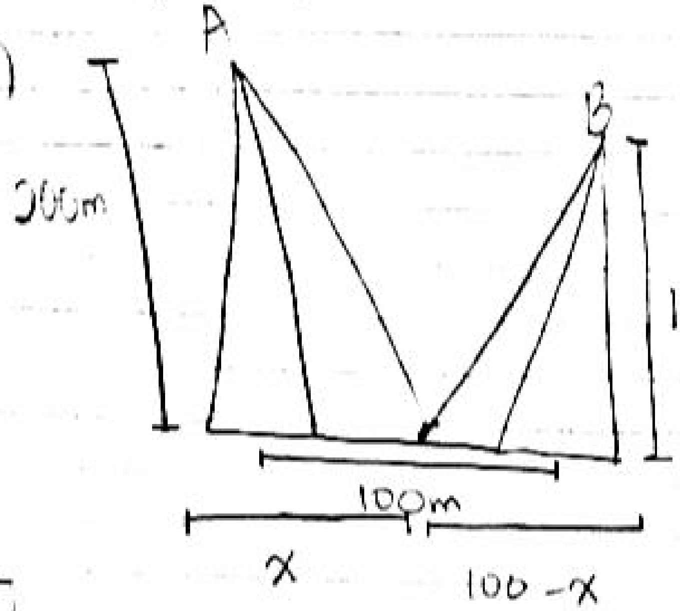
$0 \leq r \leq 6/\pi$

Endpoints: $\frac{6}{\pi}$, 0, 3 → critical values

r	A(r)	
0	9	→ min
$\frac{6}{\pi}$	11.45	
3	31.25	→ max ?

To minimize, the length of one piece
$x=0$, $y=12$ (only circle is made)

To maximize, the length of one piece is 3cm and the circle is not made, only the square is.

3) 

Minimize

$\sqrt{172^2+(100-x)^2} \leq x \leq \sqrt{\ldots}$

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Length of cable A $= \sqrt{200^2+x^2}$

Cable B Length $= \sqrt{172^2+(100-x)^2}$

Length total $= \sqrt{200^2+x^2} + \sqrt{172^2+(100-x)^2}$

$$L(x) = (40000+x^2)^{1/2} + (29584+(10000+x^2-200x))^{1/2}$$

$$L'(x) = 1/2(40000+x^2)(2x) + 1/2(39584+x^2-200x)(2x-200)$$

$L'(x)=0$	$L'(x)$ DNE
$0 = 20000+\frac{1}{2}x^2(2x)+(19792+\frac{1}{2}x^2-100x)(2x-200)$	exists everywhere
$0 = 20000+x^3+(39584+x^3-200x^2-3958400-100x^2+20000x)$	
$0 = -3898816+2x^3-300x^2+20000x$	
$3898816 = 2x(x^2-150+10000)$	
$x = 1949.408$	

Lx not possible, negative under $\sqrt{\ }$ in square root function

not in interval.
Critical values are 0, 100

x	$L(x)$
0	397.61
100	395.6 → min

Length of cable A $\rightarrow \sqrt{200^2+x^2} = \sqrt{200^2+100^2} = 223.60$

Length of cable B $\rightarrow \sqrt{172^2+(100-x)^2} = \sqrt{172^2+(0)^2} = 172$

$\therefore$, the cable should be placed at one extreme end

So the distance from cable to tower is 100m