1. Find y'. You do not need to simplify your result.

   (a) (4 marks) $y = \sec(xe^x) + 4^{e^x}$

   (b) (5 marks) $e^y \cot x = y + xy$

   (c) (5 marks) $y = x^4 \sin^{-1}(3x) + \tan^{-1}(x^3 + 2)$

   (d) (6 marks) $y = (\sin x)^{\cos^2 x}$

2. (4 marks) Show that $\dfrac{d}{dx}(\csc^{-1} x) = \dfrac{-1}{x\sqrt{x^2-1}}$.

3. (a) (5 marks) For what values of $x \in [0, 2\pi]$ does the graph of $f(x) = 4x - 3\tan x$ have a horizontal tangent?

   (b) (5 marks) At what point on the curve $y = x + \ln x$ is the tangent line parallel to the line $6x - 2y = 5$?

4. (6 marks) Find the second derivative y'' for the curve $y^3 = x + y$. Simplify your result.

$\frac{24}{30}$

1)a) $y = \sec(xe^x) + 4^{e^x}$

$y' = \left(\sec(xe^x) + 4^{e^x}\right)'$

4

$y' = (\sec(xe^x)\tan(xe^x))((1)(e^x) + (x)(e^x)) + (4^{e^x})(\ln 4)(e^x)$

$y' = \sec(xe^x)\tan(xe^x)(e^x + xe^x) + 4^{e^x}(\ln 4)(e^x)$

4/12

1b) $(e^y \cot x)' = (y + xy)'$

$(e^y)'(\cot x) + (e^y)(\cot x)' = (y)' + (xy)'$

$(e^y)(y')(\cot x) + (e^y)(\csc^2 x) = y' + y + xy'$

-1 4

$(y')(e^y)(\cot x) + (e^y)(\csc^2 x) = y' + xy' + y$

$(e^y)(\csc^2 x) = [(y')(1+x) + y][(-e^y \cot x)]$

$(e^y)(\csc^2 x) = [(y')(-e^y \cot x)[(1+x)] + y]$

$y' = \dfrac{-y + (e^y)(\csc^2 x)}{(-e^y \cot x)(1+x)}$

1c) $y' = \left(x^4 \sin^{-1}(3x) + \tan^{-1}(x^3+2)\right)'$

$y' = [x^4 \sin^{-1}(3x)]' + [\tan^{-1}(x^3+2)]'$

$y' = \left[(4x^3)(\sin^{-1}(3x)) + (x^4)\left(\frac{1}{\sqrt{1-(3x)^2}}\right)(3)\right] +$

5

$\frac{1}{1+(x^3+2)^2}(3x^2)$

$y' = (4x^3)(\sin^{-1}(3x)) + \left(3x^4 / \sqrt{1-9x^2}\right) + \frac{3x^2}{1+(x^3+2)^2}$

1d) $y = \left((\sin x)^{\cos^2 x}\right)$

-2

$\ln y = \ln\left((\sin x)^{\cos^2 x}\right)$

4

$\ln y = (\cos^2 x)(\ln(\sin x))$

$\left(\frac{1}{y}\right)(y') = (\cos^2 x)'(\ln \sin x) + \cos^2 x(\ln \sin x)'$

$[(-2\cos x \sin x)(\ln \sin x) + \frac{\cos^3 x}{\sin x}][x]$

$(y') =$

2) $\frac{d}{dx} = \csc^{-1}(x)$

$y = \csc^{-1}(x)$

$\csc(y) = \csc\left(\csc^{-1}(x)\right)$

$\csc(y) = x$

$\frac{d}{dx}(\csc y) = \frac{d}{dx}(x)$

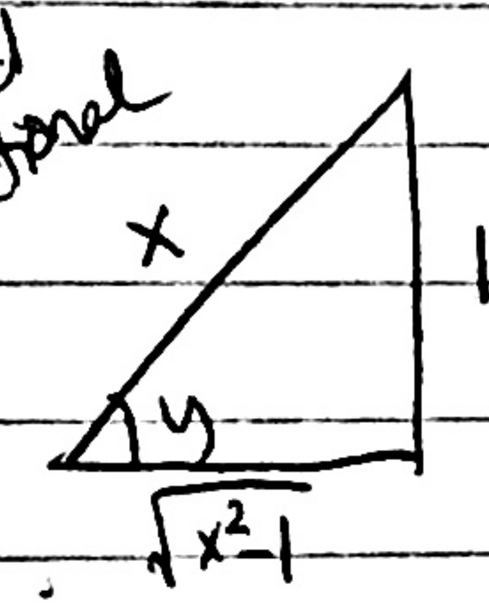
$-\csc y \cdot \cot y \cdot y' = 1$

4

$y' = \frac{-1}{\csc y \cot y}$

$y' = -\sin y \tan y$ → flip to get rid of rational

$y = -\left(\frac{1}{x}\right)\left(\frac{\sin y}{\cos y}\right)$



$\csc y = x$

$y = \frac{-1}{x} \cdot \frac{\frac{1}{x}}{\sqrt{1-\frac{1}{x^2}}}$

$\sin y = \frac{O}{H} = \frac{1}{x}$

$\tan y = \frac{O}{A} = \frac{1}{\sqrt{x^2-1}}$

$= -\frac{1}{x^2\sqrt{1-\frac{1}{x^2}}}$

$= -\frac{1}{(x)(\sqrt{x^2-1})}$

→ OR

do

$y' = -\sin y \tan y$

$= -\left(\frac{1}{x}\right)\left(\frac{1}{\sqrt{x^2-1}}\right)$

$= \frac{-1}{x\sqrt{x^2-1}}$

3a) $f(x) = 4x - 3\tan x$

$f'(x) = (4x - 3\tan x)'$

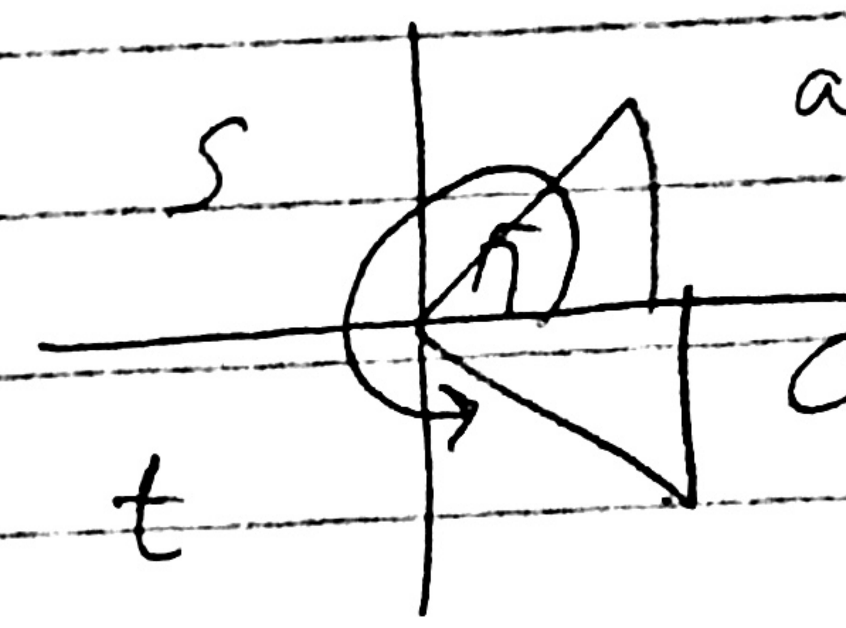
$4 - 3(\sec^2 x) = 0$

$\sec^2 x = \dfrac{-4}{-3}$

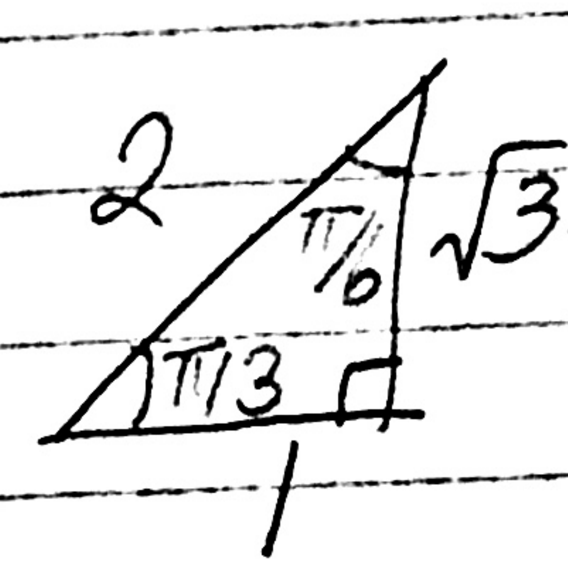
4

$\sec x = \dfrac{\sqrt{4}}{\sqrt{3}}$

$\cos x = \dfrac{\sqrt{3}}{\sqrt{4}} = \dfrac{\sqrt{3}}{2}$



$x = \pi/6, \dfrac{11\pi}{6}$ $\quad \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$



The graph of $f(x) = 4x - 3\tan x$ has a horizontal tangent at $\dfrac{\pi}{6}$ and $\dfrac{11\pi}{6}$

3b) tan line is parallel to $6x - 2y = 5$

$y = \dfrac{5 - 6x}{-2}$

$y = 3x - \dfrac{5}{2}$

Slope of tan line $= 3$

$y = x + \ln x$

$f'(x) = (x + \ln x)'$

$1 + \dfrac{1}{x} = 3$

5

$\dfrac{1}{x} = 3 - 1$

$\dfrac{1}{x} = 2$

$1 = 2x$

$x = \dfrac{1}{2}$

$f(x) = x + \ln x$

$f(1/2) = \frac{1}{2} + \ln\left(\frac{1}{2}\right)$

$= \frac{1}{2} + (-0.69)$

$= -0.193$ $\quad \boxed{(0.5, -0.193)}$ ← Point

4) $y^3 = x + y$

$y^3 - y = x$

$(y^3 - y)' = (x)'$

$(3y^2)(y') - (y') = 1$

$y'(3y^2 - 1) = 1$

$$y' = \frac{1}{3y^2 - 1}$$

$$y'' = \left(\frac{1}{3y^2-1}\right)'$$

$y'' = (3y^2 - 1)^{-1}$

$y'' = -(3y^2-1)^{-2}(6y \cdot y')$

$$y'' = -(3y^2-1)^{-2}\left(\frac{6y}{3y^2-1}\right)$$

$$y'' = \frac{-6y}{(3y^2-1)(3y^2-1)^2}$$

$$y'' = \frac{-6y}{(3y^2-1)^3}$$