Parent Signature:

Test

Unit 2 Test 3: Applying Quadratic Models (Ch. 5)

QR 43/43 100%

> I will be able to:
> - Transform the graph of $y = x^2$ when given an equation in standard, factored, or vertex form (QR2)
> - Apply my understanding of quadratic relations to a variety of problem solving situations (QR4)

<u>Instructions and Hints:</u>

- **Show your work and do your best to explain your thinking clearly! I can't give part marks when I don't know what you did!**
- **Define variables and write concluding statements when it is appropriate.**
- **<u>Use what you know</u>. There are three forms of quadratic relations. Use the most appropriate one!**
- **When graphing, be sure to include $y = x^2$. Label your final graph as well.**
- **Use fractions when necessary! *Decimals are only okay when they are given in the question*!**

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1. State the transformations that you would apply to the graph of $y = x^2$ to produce the graph of $y = -3(x+4)^2 - 9$. How many zeros do you expect the graph to have and why? (3 marks)

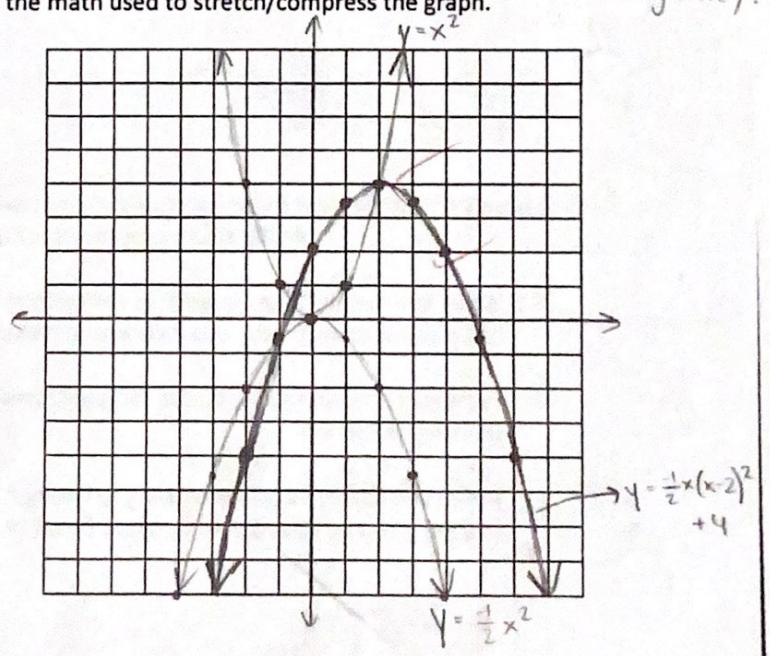
3

① Vertical stretch by a factor of 3 ③ shift parabola 4 units left
② Reflection over X-axis ④ shift parabola 9 units down

I expect the graph to have no zeros as the signs of "a" and "k" are both the same (negative).

2. Accurately sketch the graph of $y = -\frac{1}{2}(x-2)^2 + 4$ by transforming the graph of $y = x^2$. You must show a table of values for $y = x^2$ and the math used to stretch/compress the graph. (4 marks)

x	(y) $y=x^2$		(y) $y=-\frac{1}{2}x^2$
-4	16	$\times -\frac{1}{2}$ →	-8
-3	9	$\times -\frac{1}{2}$ →	-4.5
-2	4	$\times -\frac{1}{2}$ →	-2
-1	1	$\times -\frac{1}{2}$ →	-0.5
0	0	$\times -\frac{1}{2}$ →	0
1	1	$\times -\frac{1}{2}$ →	-0.5
2	4	$\times -\frac{1}{2}$ →	-2
3	9	$\times -\frac{1}{2}$ →	-4.5
4	16	$\times -\frac{1}{2}$ →	-8



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3. Write a relation in vertex form for a parabola that:

a. Has been vertically stretched by a factor of five and translated seven units right and ten units down. (2 marks)

$a = 5$
$h = 7$
$k = -10$

$\therefore y = 5(x-7)^2 - 10$

2

b. Has a vertex of (-2, 3) and passes through the point (-8, -2). (3 marks)

$x = -8$
$y = -2$
$h = -2$
$k = 3$

$y = a(x-h)^2 + k$
$-2 = a(-8+2)^2 + 3$
$-2 - 3 = a(-6)^2$
$\frac{-5}{36} = \frac{36a}{36}$
$\boxed{-5/36 = a}$

$\therefore y = \frac{-5}{36}(x+2)^2 + 3$

3

4. The graph of $y = 2(x-1)^2 + 4$ is reflected over the x-axis and translated three units left and two units up. Write the equation of the new parabola. How many zeros does the new parabola have? (3 marks)

x	$y = 2x^2$
-2	8
-1	2
0	0
1	2
2	8

The new parabola has no zeros as the signs on a and k are both the same (negative).

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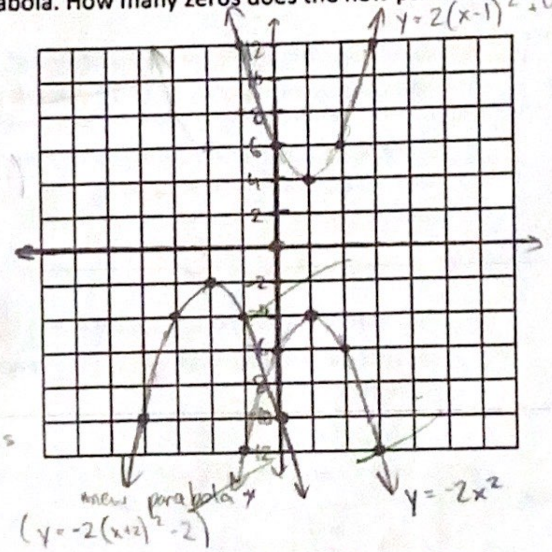
$\therefore$ the equation of the new parabola is $y = -2(x+2)^2 - 2$.



Choose TWO of the next three questions and answer them in the SPACE PROVIDED ON THE NEXT PAGE. Make sure that your explanations are clear!

5. When you are given an equation in vertex form, how can you tell how many zeros the parabola has without doing math? Use diagrams and examples to support your answer. (4 marks)

6. Can every equation be written in factored form? Include diagrams and examples to support your thinking. (4 marks)

~~7.~~ When you partial factor, what are you finding? How does this help you to find the vertex when you are given an equation in standard form? (4 marks)

(5) If you have an equation in vertex form, the signs of "a" and "k" determine the # of zeros that the parabola has.

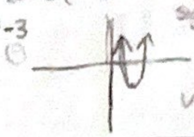
- 2 zeros → signs (of "a" and "k") are opposite (one is ⊕, one is ⊖) → • moves up, opens down • moves down, opens up (in both cases, the parabola will open in the direction of the x-axis and cross it)
- No zeros → signs are the same (both ⊖ or both ⊕)
- one zero → $k = 0$ (no k term)

* there is only one zero when $k = 0$ because "k" controls the vertical shift, ∴ the vertex will be the only zero as the parabola isn't shifted up/down

* if the signs are same it means it either opens up AND moves up, or opens down AND moves down. Both cases push the parabola away from the x-axis (∴ no zeros).

Ex: *2 zeros (a and k = opposite signs)

$y = 2(x-4)^2 - 3$
⊕ ⊖



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(6) No, every equation cannot be written in factored form as factored form equations require zeros. Not all parabolas have zeros.

* if the sign of "a" and "k" are the same, there are no zeros (∴ you can't write in FF)

(*it's also sometimes difficult to use factored form because not all relations have integers (some have long decimals))

FF format

$y = a(x-r)(x-s)$

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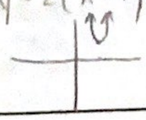
Ex: $y = 2(x-4)^2 + 3$



(*) → no zeros means there are no "r" and "s" values, which are required to write in factored form

8. Student Parliament sells hot chocolate in December to raise money for the LaSalle Food Bank. They know that they reach their maximum profit of \$1100 when they charge \$2 per hot chocolate. They also know that they make \$900 when they charge \$4 per hot chocolate. ← (vertex)

   a. Write a quadratic relation in vertex form to represent Student Parliament's profit. (3 marks)

$x = 4$
$y = 900$
$h = 2$
$k = 1100$

solve for a

$y = a(x-h)^2 + k$

$900 = a(4-2)^2 + 1100$

$900 - 1100 = a(2)^2$

$\frac{-200}{4} = \frac{4a}{4}$

$\boxed{-50 = a}$

3

$\therefore y = -50(x-2)^2 + 1100$ (vertex form equation)

   b. Write the relation from a. in standard form. (3 marks)

$y = -50(x-2)^2 + 1100$

$y = -50(x-2)(x-2) + 1100$

$y = -50(x^2 - 2x - 2x + 4) + 1100$

$y = -50(x^2 - 4x + 4) + 1100$

$y = -50(x^2 - 4x + 4) + 1100$

$y = -50x^2 + 200x - 200 + 1100$

$y = -50x^2 + 200x + 900$

3

∴ the relation in standard form is "$y = -50x^2 + 200x + 900$".

9. Write $y = -3x^2 + 15x + 2$ in vertex form. Remember, answers should be fractions not decimals! (3 marks)

partial factor w/ shortcut

$a = -3$
$b = 15$
$c = 2$

$X = \frac{-b}{2a}$

3

$X = \frac{-15}{2(-3)}$

$X = \frac{-15}{-6}$

A of S → $\boxed{X = \frac{5}{2}}$
(plug into equation)

$y = -3\left(\frac{5}{2}\right)^2 + 15\left(\frac{5}{2}\right) + 2$

$y = -3\left(\frac{25}{4}\right) + \frac{75}{2} + 2$

$y = \frac{-75}{4} + \frac{150}{4} + \frac{8}{4}$

$\boxed{y = \frac{83}{4}}$

∴ Vertex $\left(\frac{5}{2}, \frac{83}{4}\right)$

$\therefore y = -3\left(x - \frac{5}{2}\right)^2 + \frac{83}{4}$ (vertex form)

CHOOSE EITHER #10 OR #11 (4 marks)

10. The cost, C, in dollars per hour of running a steamboat is modelled by the quadratic function $C = 1.8x^2 - 11.52x + 126.2$, where x is the speed in kilometers per hour. At what speed should the boat travel to minimize cost? What is the minimum cost?

~~11. The profit, P, for a cosmetics company, in thousands of dollars, is given by $P = -5x^2 + 400x - 2200$, where x is the amount spent on advertising in thousands of dollars.~~ Determine the maximum profit, and the amount that should be spent on advertising to maximize profit.

10) * partial factor to find A of S *

a = 1.8
b = -11.52
c = 126.2

4

$X = \frac{-b}{2a}$

$X = \frac{11.52}{2(1.8)}$

$X = \frac{11.52}{3.6}$

$\boxed{X = 3.2}$

∴ the boat should travel at 3.2 km/h to minimize cost.

* use A of S to find min. cost *

$C = 1.8(3.2)^2 - 11.52(3.2) + 126.2$

$C = 1.8(10.24) - 36.864 + 126.2$

$C = 18.432 - 36.864 + 126.2$

$C = 107.768$ (~\$107.77)

∴ the minimum cost per hour of running the steamboat is \$107.77

12. Find the x-intercepts (zeros) for $y = -3(x-2)^2 + 27$ **without expanding**. Rearrange to isolate x! (3 marks)

$y = -3(x-2)^2 + 27$

$0 = -3(x-2)^2 + 27$

$\frac{-27}{-3} = \frac{-3(x-2)^2}{-3}$

$\sqrt{9} = (x-2)^2$

3

$\pm 3 = x - 2$

$\pm 3 = x - 2$

$3 = x - 2$ $\quad$ $-3 = x - 2$

$3 + 2 = x$ $\quad$ $-3 + 2 = x$

$\boxed{5 = x}$ $\quad$ $\boxed{-1 = x}$

$(5, 0)$ $\quad$ $(-1, 0)$

∴ the zeros of the given equation are at (5, 0) and (-1, 0).

13. Determine the exact values of a and b in the relation y = ax² + bx + 6 if the vertex is located at (2, 4). (4 marks)

y-intercept (0, 6)

Solve for "a" first

x = 0
y = 6
h = 2
k = 4

$y = a(x-h)^2 + k$

$6 = a(0-2)^2 + 4$

$6 - 4 = a(-2)^2$

$\frac{2}{4} = \frac{4a}{4}$

$\boxed{\frac{1}{2} = a}$

$\therefore y = \frac{1}{2}(x-2)^2 + 4$ (vertex form)

4

* expand/simplify to find b *

$y = \frac{1}{2}(x-2)^2 + 4$

$y = \frac{1}{2}(x-2)(x-2) + 4$

$y = \frac{1}{2}(x^2 - 2x - 2x + 4) + 4$

$y = \frac{1}{2}(x^2 - 4x + 4) + 4$

$y = \frac{1}{2}x^2 - \frac{4}{2}x + \frac{4}{2} + 4$

$y = \frac{1}{2}x^2 - 2x + 2 + 4$

$y = \frac{1}{2}x^2 - 2x + 6$

$\boxed{\therefore b = -2}$

$\therefore a = \frac{1}{2}$ and $b = -2$.