30.5/40

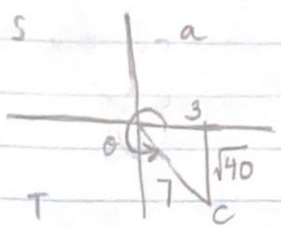
Department of Mathematics and Statistics

Differential Calculus

Lab Assignment 2

1. (5 marks) If $\cos\theta = \frac{3}{7}$, $\pi < \theta < 2\pi$, find the remaining trigonometric ratios.
2. Prove the following identities
   (a) (6 marks) $\cos 3x = 4\cos^3 x - 3\cos x$
   (b) (6 marks) $\cot x + \cot y = \dfrac{\sin(x+y)}{\sin x \sin y}$
3. If $\sin x = -\frac{1}{2}$ and $\tan y = \frac{3}{4}$, where x and y are in the interval $[\pi/2, 3\pi/2]$, evaluate the expression.
   (a) (6 marks) $\cos(2x - 2y)$
   (b) (6 marks) $\tan(x + y)$
4. Find all the values of x in the interval $[0, 2\pi]$ that satisfy the equation.
   (a) (5 marks) $3\tan^2 x = 1$
   (b) (6 marks) $\cos 2x = 1 + 3\cos x$

1) $\cos\theta = \frac{3}{7}$



$\sqrt{7^2-3^2} = \sqrt{49-9} = \sqrt{40}$

$\sin\theta = -\sqrt{40}/7$ ✓ SOH CAH TOA

$\tan\theta = -\sqrt{40}/3$ ✓

$\csc\theta = -7/\sqrt{40}$ ✓

$\sec\theta = 7/3$ ✓

$\cot\theta = -3/\sqrt{40}$ ✓

5 1/4

2) a) $\cos 3x = 4\cos^3 x - 3\cos x$

LS	RS
$\cos 3x$	$4\cos^3 x - 3\cos x$

$\cos(a+b)$

$= \cos(x+2x)$ ✓

$= \cos x\cos 2x - \sin x\sin 2x$ ✓

$= \cos x\cos(x+x) - \sin x\sin(x+x)$

$= \cos x\cos x\cos x + \sin x\sin x\cos x - \sin x\sin x\cos x + \cos x\sin x\sin x$

$= \cos^3 x + \sin^2 x\cos x - \sin^2 x\cos x + \cos x\sin^2 x$

$= \cos^3 x + (1-\cos^2 x)\cos x - (1-\cos^2 x)\cos x + \cos x\sin^2 x$

$\therefore \cos^3 x + \cos x - \cos^3 x + \cos x - \cos^3 x + \cos x + \cos^3 x$

5 1/6 $= 4\cos^3 x - 3\cos x$ ✓

$=$

2b) $\cot x + \cot y = \frac{\sin(x+y)}{\sin x\sin y}$

LS	RS
$\cot x + \cot y$	$\frac{\sin(x+y)}{\sin x\sin y}$

3 1/6 $= \frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}$ ✓

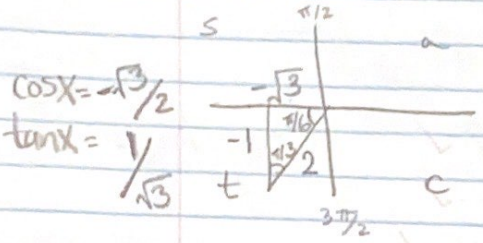
$= \frac{\cos x\sin y + \cos y\sin x}{\sin x\sin y}$ ✓

$= \frac{\sin(x+y)}{\sin x\sin y}$

$\therefore$ LS = RS

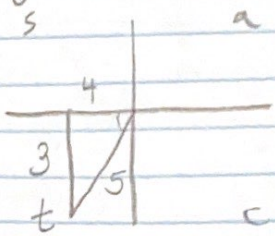
Hilroy

3) $\sin X = -1/2$ $\quad \tan y = 3/4$



$\cos X = -\sqrt{3}/2$

$\tan X = 1/\sqrt{3}$



$\cos y = -4/5$

$\sin y = -3/5$

3a) $\cos(2x-2y)$ ✓

$= \cos 2x \cos 2y + \sin 2x \sin 2y$ ✓

$= (\cos^2 X - \sin^2 X)(\cos^2 y - \sin^2 y) + (2\sin X \cos X)(2\sin y \cos y)$ ✓

$= \left((-\sqrt{3}/2)^2 - (-1/2)^2\right)\left((-4/5)^2 - (-3/5)^2\right) + \left(2(-1/2)(-\sqrt{3}/2)\right)\left(2(-\frac{3}{5})(-\right.$

5.5/6 $= \left[\left(\frac{3}{4}\right) - \left(\frac{1}{4}\right)\right]\left[\frac{16}{25} - \frac{9}{25}\right] + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{24}{25}\right)$ ✓

$= \left(\frac{1}{2}\right)\left(\frac{7}{25}\right) + \frac{24\sqrt{3}}{50} = \frac{7+24\sqrt{3}}{50}$ ✓

3b) $\tan(x+y) = \dfrac{\tan X + \tan y}{1 - \tan X \tan y} = \dfrac{\left(1/\sqrt{3}\right) + \left(3/4\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)\left(\frac{3}{4}\right)}$ ✓

6/6 $\tan X = 1/\sqrt{3}$ ✓

$\tan y = 3/4$ ✓

$= \dfrac{4+3\sqrt{3}}{4\sqrt{3}} \div \left(1 - \dfrac{3}{4\sqrt{3}}\right)$ ✓

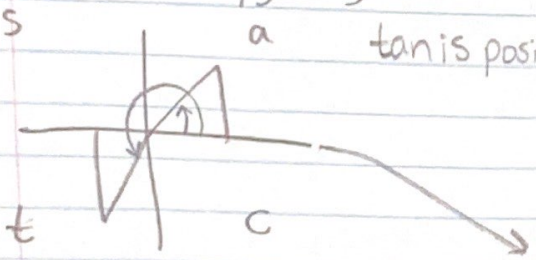
$= \dfrac{4+3\sqrt{3}}{4\sqrt{3}} \div \left(\dfrac{4\sqrt{3}-3}{4\sqrt{3}}\right)$ ✓

$= \dfrac{4+3\sqrt{3}}{4\sqrt{3}} \times \dfrac{4\sqrt{3}}{4\sqrt{3}-3}$

$= \dfrac{4+3\sqrt{3}}{4\sqrt{3}-3}$ ✓

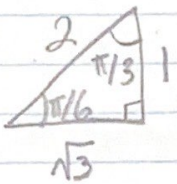
4) a) $3\tan^2 X = 1$

$\tan^2 X = 1/3$ → $\tan X = \sqrt{1/3} = \pm\frac{1}{\sqrt{3}}$ ✓



tan is positive in quadrants 1 and 3

Special triangle



$X = \pi/6, \pi + \pi/6 = 7\pi/6$

$\therefore X = \frac{\pi}{6}, \frac{7\pi}{6}$ ✓ ✓

4b) $\cos 2x = 1 + 3\cos x$

$\cos 2x - 3\cos x = 1$ ✓

$(2\cos^2 X - 1) - 3\cos X = 1$ ✓

$2\cos^2 X - 3\cos X - 2 = 0$

Let $\cos X = x$ ✓

Rewrite → $2x^2 - 3x - 2 = 0$

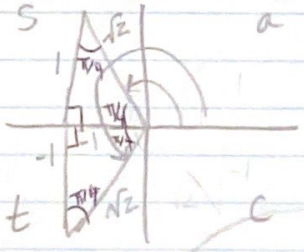
$\frac{3 \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$

NOT → 2 , $-1/2$

Now, sub $\cos X$ back for x.

$\cos X = -1/2$ → $X = 3\pi/4, 5\pi/4$ ✓

Special triangle:



$\therefore X = \frac{3\pi}{4}, \frac{5\pi}{4}$