35/35

Differential Calculus

Lab Assignment 5

Show All your work to receive Full credit.

1. Evaluate the following limits if they exist.

   (a) (2 marks) $\lim_{x\to 5} \frac{x^2-3x+1}{x^3+x+1}$.

   (b) (3 marks) $\lim_{x\to -3} \frac{x^2+2x-3}{x^2+8x+15}$.

   (c) (3 marks) $\lim_{h\to 0} \frac{(h+4)^2-16}{h}$.

   (d) (5 marks) $\lim_{h\to 0} \frac{\sqrt{h+5}-\sqrt{5}}{h}$

   (e) (5 marks) $\lim_{x\to 8} \frac{|x-8|}{x^2-64}$

   (f) (4 marks) $\lim_{x\to 0} \frac{x}{\tan(5x)}$

   (g) (5 marks) $\lim_{x\to\infty} \left(\sqrt{x^2+x}-x\right)$ → rationalize

   (h) (4 marks) $\lim_{x\to -\infty} \frac{\sqrt{x^2-x}-x}{x+2}$ → don't rationalize b/c you have a denominator

   (i) (4 marks) $\lim_{x\to 0^+} x^6 \cos\ln x$

1a) $\lim_{x\to 5} \dfrac{x^2-3x+1}{x^3+x+1}$

$= \dfrac{(5)^2-3(5)+1}{(5)^3+(5)+1} = \boxed{\dfrac{11}{131}}$

1b) Cannot sub in or denominator equals zero

3

$\lim_{x\to -3} \dfrac{(x-1)\cancel{(x+3)}}{\cancel{(x+3)}(x+5)} = \lim_{x\to -3} \dfrac{(x-1)}{(x+5)} = \dfrac{(-3-1)}{(-3+5)} = \dfrac{-4}{2} = \boxed{-2}$

1c) $\lim_{h\to 0} \dfrac{(h+4)^2-16}{h} = \lim_{h\to 0} \dfrac{(\cancel{h+4-4})(h+4+4)}{\cancel{h}}$

3

$= \lim_{h\to 0} (h+8) = (0+8) = \boxed{8}$

1d) $\lim_{h\to 0} \dfrac{\sqrt{h+5}-\sqrt{5}}{h} \times \dfrac{\sqrt{h+5}+\sqrt{5}}{\sqrt{h+5}+\sqrt{5}}$

5

$= \lim_{h\to 0} \dfrac{(\sqrt{h+5})^2-(\sqrt{5})^2}{h(\sqrt{h+5}+\sqrt{5})} = \lim_{h\to 0} \dfrac{\cancel{h}+\cancel{5-5}}{\cancel{h}(\sqrt{h+5}+\sqrt{5})} = \lim_{h\to 0} \dfrac{1}{\sqrt{h+5}+\sqrt{5}}$

$= \dfrac{1}{\sqrt{0+5}+\sqrt{5}} = \boxed{\dfrac{1}{2\sqrt{5}}}$

1e) $\lim_{x\to 8} \dfrac{|x-8|}{x^2-64}$

5

$\lim_{x\to 8^+} \dfrac{-(x-8)}{x^2-64}$

$= \lim_{x\to 8^+} \dfrac{-\cancel{(x-8)}}{\cancel{(x-8)}(x+8)}$

$= \lim_{x\to 8^+} \dfrac{-1}{(x+8)}$

$= \dfrac{-1}{(8+8)} = \dfrac{-1}{16}$ → Left hand limit

$\lim_{x\to 8^-} \dfrac{(x-8)}{x^2-64}$

$= \lim_{x\to 8^-} \dfrac{\cancel{(x-8)}}{\cancel{(x-8)}(x+8)}$

$= \lim_{x\to 8^-} \dfrac{1}{x+8}$

$= \dfrac{1}{8+8} = \dfrac{1}{16}$ → right hand limit

LHL ≠ RHL

$\boxed{\therefore \text{, limit doesn't exist}}$

f) $\lim\limits_{x\to 0} \dfrac{x}{\tan(5x)} = \lim\limits_{x\to 0} \dfrac{x}{\frac{5\tan(5x)}{5x}} = \dfrac{1}{5} \lim\limits_{x\to 0} \dfrac{1}{\frac{\tan 5x}{5x}}$

4

$= \dfrac{1}{5} \dfrac{\lim\limits_{x\to 0} 1}{\lim\limits_{x\to 0} \frac{\tan 5x}{5x}} = \left(\dfrac{1}{5}\right)\left(\dfrac{1}{\lim\limits_{5x\to 0} \frac{\tan 5x}{5x}}\right) = \dfrac{1}{5}\left(\dfrac{1}{1}\right) = \boxed{\dfrac{1}{5}}$

1g) $\lim\limits_{x\to\infty} \sqrt{x^2+x} - x \cdot \dfrac{\sqrt{x^2+x} + x}{\sqrt{x^2+x} + x}$

5

$= \lim\limits_{x\to\infty} \dfrac{\left(\sqrt{x^2+x}\right)^2 - x^2}{\sqrt{x^2+x} + x} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2} + x - \cancel{x^2}}{\sqrt{x^2+x} + x}$

$= \lim\limits_{x\to\infty} \dfrac{x}{\sqrt{x^2+x} + x} = \lim\limits_{x\to\infty} \dfrac{x/x}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2}} + \frac{x}{x}}$

$= \lim\limits_{x\to\infty} \dfrac{1}{\sqrt{1+\frac{1}{x}} + 1} = \lim\limits_{x\to\infty} \dfrac{1}{\sqrt{1+0} + 1} = \boxed{\dfrac{1}{2}}$

1h) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2-x} - x}{x+2} = \lim\limits_{x\to-\infty} \dfrac{\frac{\sqrt{x^2-x}}{x} - \frac{x}{x}}{\frac{x}{x} + \frac{2}{x}}$

4

$= \lim\limits_{x\to-\infty} \dfrac{\frac{\sqrt{x^2-x}}{\sqrt{x^2}} - \frac{x}{x}}{\frac{x}{x} + \frac{2}{x}} = \lim\limits_{x\to-\infty} \dfrac{\sqrt{\frac{x^2-x}{x^2}} - \frac{x}{x}}{\frac{x}{x} + \frac{2}{x}}$

$= \lim\limits_{x\to-\infty} \dfrac{\sqrt{1-\frac{1}{x}} - 1}{1+2/x} = \lim\limits_{x\to-\infty} \dfrac{-\sqrt{1-0} - 1}{1+0} = \lim\limits_{x\to-\infty} \dfrac{-1-1}{1} = \boxed{-2}$

) $\lim_{x \to 0^+} x^6 \cos \ln x$

$-1 \leq \cos(\ln x) \leq 1$

4 $-x^6 \leq x^6 \cos(\ln x) \leq x^6$

$\lim_{x \to 0^+} (-x^6) \leq \lim_{x \to 0^+} x^6 \cos(\ln x) \leq \lim_{x \to 0^+} (x^6)$

$0 \leq \lim_{x \to 0^+} x^6 \cos(\ln x) \leq 0$

$\therefore \lim_{x \to 0^+} x^6 \cos(\ln x) = \boxed{0}$