25/25

Differential Calculus

Lab Assignment 9

Show All your work to receive Full credit.

1. (5 marks) Use the definitions of hyperbolic functions to show that $(\text{sech } x)' = \text{sech } x \tanh x$
2. (5 marks) Use a linear approximation to estimate $(1.02)^{1/3}$.
3. (5 marksa) Two sides of a triangle are 3 m and 7 m in length and the angle between them is increasing at a rate of 0.05 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/6$.
4. (5 marks) A 1.4 m tall woman is walking away from a 2.8 m tall lamp post. If she is walking at a rate of 1.2 m/s, at what rate is the length of her shadow increasing?
5. (5 marks) Find the absolute maximum and absolute minimum values of the function $f(x) = x - 2\tan^{-1} x$, on the interval $[0, 4]$.

1) $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$

Definition is $(\operatorname{sech} x) = \dfrac{2}{e^x + e^{-x}}$

$$(\operatorname{sech} x)' = \left[\frac{1 \to f(x)}{(\cosh x) \to g(x)}\right]'$$

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Quotient Rule $\to$ $\dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$$= \frac{(1)'(\cosh x) - (1)(\cosh x)'}{(\cosh x)^2}$$

$$= \frac{-(\cosh x)'}{(\cosh x)^2} = -\left(\frac{e^x + e^{-x}}{2}\right) \cdot \frac{1}{(\cosh x)^2}$$

$$= -(\sinh x) \cdot (\operatorname{sech} x)^2$$

$$= -\frac{\sinh x}{\cosh x} \cdot \frac{1}{\cosh x}$$

$$\boxed{= -\tanh x \cdot \operatorname{sech} x}$$

2) $f(x) = x^{1/3} = \sqrt[3]{x}$

$a = 1 \to$ Point is $(1,1) \to$ tangent passes through this

5

$$f'(x) = (x^{1/3})'$$

$$= \frac{1}{3} x^{-2/3}$$

$m =$ $f'(1) = \dfrac{1}{3}(1)^{-2/3} = \dfrac{1}{3}$

$$y - 1 = \frac{1}{3}(x - 1)$$

$$y = \frac{1}{3}x - \frac{1}{3} + 1$$

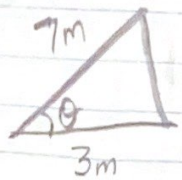
$$y = \frac{1}{3}x + \frac{2}{3}$$

$$L(x) = \frac{1}{3}x + \frac{2}{3}$$

$$L(1.02) = \frac{1}{3}(1.02) + \frac{2}{3}$$

$\to L(1.02) = 1.006$

3)



θ is increasing at a rate of 0.05 rad/s when $\theta = \frac{\pi}{6}$, find the rate at which area of triangle is increasing.

2 quantities: angle θ, area of triangle
↳ let this be θ ↳ let this be A

$\sin\theta = \frac{h}{7} \rightarrow h = 7\sin\theta$

5

$A = \frac{1}{2}bh = \frac{1}{2}b(7\sin\theta)$

$A' = \frac{7}{2}\left[\cancel{3' \sin\frac{\pi}{6}} + 3\cos\frac{\pi}{6}(\theta')\right]$

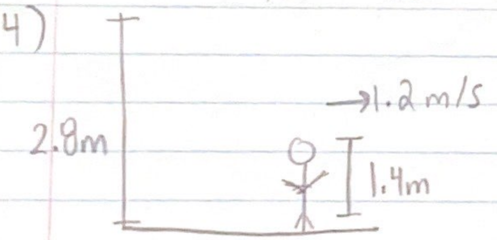
$A' = \frac{7}{2}\left[3\cos\frac{\pi}{6}(0.05)\right]$

$A' = \frac{21}{2} \times \frac{\sqrt{3}}{2} \times 0.05 = \frac{1.05\sqrt{3}}{4}$

$\frac{21\sqrt{3}}{80}$

∴, the area of the triangle is increasing at a rate of $\frac{1.05\sqrt{3}}{4}$, or 0.45, rad/s

4)



2 quantities → distance to post is x
→ length of shadow is l

5

$\frac{2.8}{x+l} = \frac{1.4}{l}$

$2.8l = (x+l)(1.4)$

$2.8l - 1.4l = 1.4x$

$1.4l = 1.4x$

$x = l$

$(x)' = (l)'$

$l' = (1.2)$

$l' = 1.2\ \cancel{ft}\ m/s$

∴, her shadow is increasing at a rate of 1.2 m/s

5) $f(x) = x - 2\tan^{-1}x$

$f'(x) = (x - 2\tan^{-1}x)'$

$= 1 - 2(1/1+x^2)$

$= 1 - \dfrac{2}{1+x^2}$

5

$f'(x)=0$	$f'(x)$ DNE (doesn't exist)
$1 - \dfrac{2}{1+x^2} = 0$	$1+x^2 = 0$
$-1 = \dfrac{-2}{1+x^2}$	$-1 = x^2$
$\dfrac{-2}{-1} = 1+x^2$	$\sqrt{-1} = x$
$2 = 1+x^2$	Not possible
$1 = x^2$	f(x) exists everywhere
$x = \pm 1$	

$x = 1$ and $x = \cancel{-1}$ not in [0,4] are critical numbers in the interval [0,4]

Evaluate endpoints and critical values → 1, −1, 0, 4

x	f(x)	
$\cancel{-1}$	$0.\cancel{57}$	
0	0	
1	−0.57	→ absolute min
4	1.34	→ absolute max

∴, when $x=1$ the graph of f(x) has an absolute min. of −0.57, and when $x=4$ the graph has an absolute max of 1.34