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DEPARTMENT OF MATHEMATICS AND STATISTICS

# Differential Calculus

## Test 2

**Instructions** :

- This test has 7 questions and a total of 6 pages. You have 80 minutes.
- Read carefully and answer **all** questions. Show all the work.
- Only non-graphing and non-programmable calculators are permitted. No cell phones or any other electronic devices are allowed.
- Work all problems in the space provided.
- Do not detach any pages.

what if the function is not continuous on one section

(4) 1. Show that the following equation has a real root in the interval $(0, 1)$.

$$8\sin^{-1}x + \ln(x+1) = x + 2$$

IVT

Let $f(x) = 8\sin^{-1}(x) + \ln(x+1) - (x+2)$

$\sin^{-1}x$ is continuous on $[-1,1]$, $8\sin^{-1}(x)$ on $[8,8]$

$\ln(x+1)$ is continuous on $(-1,\infty]$

$-(x+2)$ is continuous on all real numbers

Show <0 or >0

Guess and check

radians

$f(1) = 8\sin^{-1}(1) + \ln(2) - (3)$

$= 10.25 \rightarrow L > 0$

$f(0) = 8\sin^{-1}(0) + \ln(1) - 3$

$= -3 \rightarrow L < 0$

∴, the function $f(x)$ has a real root on the interval $(0,1)$

(6) 2. Use the definition of derivative to find the derivative of $f(x) = x + \sin x$.

$(1 + \cos x)$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h) + \sin(x+h)] - [x + \sin x]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x} + h + (\sin x \cosh + \cos x \sinh) - \cancel{x} - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{h + \sin x \cosh + \cos x \sinh - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{h}{h} + \underbrace{\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}}$$

function $(\sin x)'$

$$= \lim_{h\to 0} 1 + \cos x$$

how did you get this?

Show more steps

-2

(17) 3. Find $y'$. You do not have to simplify your answer.

$(\cot x)' = -\csc^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc x)' = \cancel{\csc}$ $-\csc\cot$

(a) (4 marks) $y = \dfrac{3^{5x}}{x^2 + \cot x}$ $\rightarrow f(x)$ $\rightarrow g(x)$

$$y' = \frac{(3^{5x})'(x^2 + \cot x) - (3^{5x})(x^2 + \cot x)'}{(x^2 + \cot x)^2}$$

$$y' = \frac{(3^{5x})(\ln 3)(5)(x^2 + \cot x) - (3^{5x})(2x - \csc^2 x)}{(x^2 + \cot x)^2}$$

4

(b) (5 marks) $\tan y + 4x^3y^2 = y^3 + x$

$(\tan y + 4x^3y^2)' = (y^3 + x)'$

$[(\sec^2 y \cdot y') + 4(x^3y^2)' = (3y^2)(y') + 1$

$[(\sec^2 y \cdot y') + 4(3x^2y^2 + x^3 2y \cdot y')] = 3y^2 y' + 1$

$[\sec^2 y \cdot y'] + [12x^2y^2 + 4x^3 \cdot 2y \cdot y'] = 3y^2 \cdot y' + 1$

$\sec^2 y \cdot y' + 8x^3 y \cdot y' - (3y^2 \cdot y') = 1 - 12x^2y^2$

$y'(\sec^2 y + 8x^3 y - 3y^2) = 1 - 12x^2y^2$

$$y' = \frac{1 - 12x^2y^2}{\sec^2 y + 8x^3 y - 3y^2}$$

5

$(\csc^{-1}x)' = \frac{-1}{|x|\sqrt{x^2-1}}$

$(\log_b x)' = (1/\ln b \cdot x)(x)'$

(c) (3 marks) $y = 3\csc^{-1}(x) + \log_3(x^2+1)$.

$$y' = \left(3\csc^{-1}(x) + \log_3(x^2+1)\right)'$$

$$y' = 3(\csc^{-1}(x))' + \frac{2x}{\ln 3 \cdot (x^2+1)}$$

3

$$y' = 3\left(\frac{-1}{|x|\sqrt{x^2-1}}\right) + \frac{2x}{\ln 3(x^2+1)}$$

$$y' = \frac{-3}{|x|\sqrt{x^2-1}} + \frac{2x}{\ln 3(x^2+1)}$$

(d) (5 marks) $y = (\sin x)^{4x}$.

$$\ln y = 4x \cdot \ln(\sin x)$$

$$(\ln y)' = (4x \cdot \ln(\sin x))' \rightarrow$$

4

$$(\ln y)' = ((4x)(\ln(\sin x)))'$$

$$(\ln y)' = (4x)'(\ln(\sin x)) + 4x \cdot (\ln(\sin x))'$$

$$= \frac{y'}{y} = 4 \cdot (\ln(\sin x)) + 4x\left(\frac{1}{\sin x} \cdot \cos x\right)$$

$$\frac{y'}{y} = 4(\ln(\sin x)) + \frac{4x\cos x}{\sin x}$$

$$y' = \boxed{y}\left(4(\ln(\sin x)) + 4x \cdot \cot x\right)$$

cannot leave in terms of y. Plug in $y = (\sin x)^{4x}$

(5) 4. Use a linear approximation to estimate $(64.04)^{1/6}$

Let $f(x) = (x)^{1/6} = \sqrt[6]{x}$

$x = 64$, $y = \sqrt[6]{64} = 2$ Point $\rightarrow (64, 2)$

$$(x^{1/6})' = \frac{1}{6}x^{-5/6} \rightarrow f'(64) = \left(\frac{1}{6(\sqrt[6]{64^5})}\right) = \frac{1}{6 \cdot 32} = \frac{1}{192}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{192}(x - 64)$$

$$y = \frac{1}{192}x - \frac{64}{192} + \frac{384}{192}$$

$$y = \frac{1}{192}x + \frac{320}{192}$$

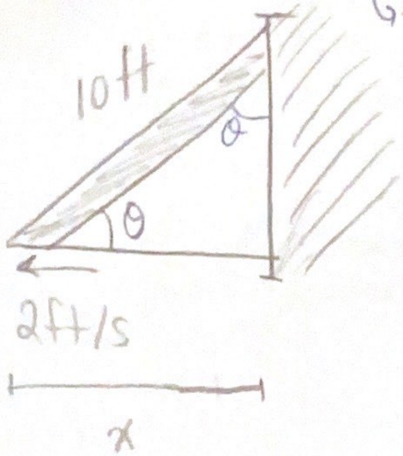
$$L(x) = \frac{1}{192}x + \frac{5}{3}$$

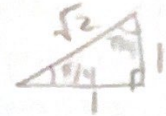
$$L(64.04) = \frac{1}{192}(64.04) + \frac{5}{3} = 2.000$$

(5) 5. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ rad?

4

+




$\theta = \frac{\pi}{4}$ rad

$x \Rightarrow$ ladder distance from wall

$x' = 2$ ft/s

$\theta' = ?$

$\cos\theta = \frac{x}{10}$

$(\cos\theta)' = \left(\frac{x}{10}\right)'$

$-\sin\theta \cdot \theta' = \frac{1}{10} x'$

$-\sin\left(\frac{\pi}{4}\right) \cdot \theta' = \frac{1}{10}(2)$

$-\left(\frac{1}{\sqrt{2}}\right) \cdot \theta' = \frac{2}{10}$

$\theta' = \frac{1}{5} \div \left(-\frac{1}{\sqrt{2}}\right)$

$\theta' = \frac{1}{5} \cdot \left(-\frac{\sqrt{2}}{1}\right)$ $+\sqrt{2}/5$

$\theta' = \left(\frac{-\sqrt{2}}{5}\right)$

∴, the angle is changing at a rate of $\frac{-\sqrt{2}}{5}$ rad/Second

$(\sin\theta)' = \left(\frac{x}{10}\right)'$

$\theta' \cdot \cos\theta = \frac{x'}{10}$

$\theta' = \frac{2}{10\cos\pi/4} \rightarrow \sqrt{2}/5$ rad/s

(6) ✓ 6. Find the absolute maximum and absolute minimum values of $f(x) = 5x^{4/5} - 4x$, on the interval $[-1, 2]$.

$f(x)\ 5x^{4/5} - 4x$

$f'(x) = (5x^{4/5} - 4x)' = 5(x^{4/5})' - 4 = 5\left(\frac{4}{5}x^{-1/5}\right) - 4 = 4x^{-1/5} - 4$

| $f'(x) = 0$ | $f'(x)$ does not exist |
|---|---|
| $\frac{4}{x^{1/5}} - 4 = 0$ <br> $\frac{4}{\sqrt[5]{x}} = 4$ <br> $1 = \sqrt[5]{x}$ <br> $x = 1$ | $\frac{4}{\sqrt[5]{x}}$ <br> $x \neq 0$ or $f'(x)$ DNE |

| Critical values/ endpoints | f(x) = ? | |
|---|---|---|
| -1 | 9 | → max |
| 0 | 0 | → min |
| 1 | 1 | |
| 2 | 0.70 | |

✓

Critical values → 1 and 0 (in interval)

Endpoints → -1 and 2

Absolute max → (-1, 9)

Absolute min → (0, 0)

(7) 7. Indicate whether each statement is True or False.

(a) ( T / F ) If $f$ is differentiable at $x = a$, then $f$ is continuous at $x = a$.

(b) ( T / F ) If $f$ is differentiable at $x = a$, then $|f|$ is also differentiable at $x = a$.

(c) ( T / F ) If $f'(c) = 0$, then $f$ has a local maximum or a local minimum at $c$. → Inflection points also have derivative = 0

(d) ( T / F) If $f$ and $g$ are continuous at $x = a$ then $fg$ is continuous at $x = a$.

(e) ( T / F ) $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)}{g'(x)}$.

(f) ( T / F ) Every rational function is continuous on its domain. → domain is where a function is defined

so like $\frac{1}{x}$ → the asymptote isn't even in the domain

(g) ( T / F ) $(\cosh x)' = \sinh x$

| Question | Points | Score |
|---|---|---|
| 1 | 4 | 4 |
| 2 | 6 | 4 |
| 3 | 17 | 16 |
| 4 | 5 | 5 |
| 5 | 5 | 4 |
| 6 | 6 | 6 |
| 7 | 7 | 5 |
| Total: | 50 | 44 |

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