Parent Signature:

Test

Chapter 4: Factoring Algebraic Expressions

QR 44.5/45 99%

Learning Goals:

- I will be able to solve quadratic equations and interpret their solutions. (QR3)
- I will be able to apply my understanding of quadratic relations to a variety of problem solving situations. (QR4)

Instructions and Hints:

- Read all questions carefully and be sure to answer all parts! Don't lose marks for not reading questions.
- ***ALWAYS look for a common factor first!!***
- Remember that you can check your answers after you factor. If you multiply it out and it isn't the same as the question, something went wrong!
- Be careful with your signs when you are finding zeros. ***SET EACH FACTOR EQUAL TO ZERO!***
- Make sure that all answers are organized and legible. If I can't follow or can't read it I am not marking it. **YOU MUST HAVE EQUALS SIGNS IN APPROPRIATE PLACES!!!**
- Remember that you need to know three points to find an equation or to sketch a graph!

Part 1: Factoring (15 marks)

1. Factor the following expressions using decomposition. (6 marks)

   a. $4t^2 - 5t - 6$ $ac = -24$ $b = -5$

   $= 4t^2 - 8t + 3t - 6$

   $= 4t(t-2) + 3(t-2)$

   $= (4t+3)(t-2)$

   6

   b. $7a^2 + 19a - 6$ $ac = -42$ $b = 19$

   $= 7a^2 + 21a - 2a - 6$

   $= 7a(a+3) - 2(a+3)$

   $= (7a-2)(a+3)$

2. Factor each of the following using a method of your choice. (9 marks)

   a. $x^2 - 5x - 24$ $ac = -24$ $b = -5$

   $= x^2 - 8x + 3x - 24$

   $= (x-8)(x+3)$

   b. $49x^2 - 225$ *diff of squares*

   $= (7x+15)(7x-15)$

   c. $4x^2 - 20x + 25$ $b = 2\sqrt{ac}$

   $= (2x-5)^2$

   d. $27x^2 - 3$

   $= 3(9x^2 - 1) \rightarrow$ (diff of squares)

   $= 3(3x-1)(3x+1)$

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Part 2: Applying Factoring Skills (20 marks)

1. Use the expression from question 2a. in part 1, $x^2 - 5x - 24$, to:

   a. Write the relation as an equation in factored form (y = your factors). (1 mark)

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$x^2 - 5x - 24$
$= x^2 - 8x + 3x - 24$ $\longrightarrow$ $y = (x-8)(x+3)$

   b. Find the zeros and the equation of the axis of symmetry. (3 marks)

Zeros:

① $x - 8 = 0$, $x = 8$, $(8,0)$

② $x + 3 = 0$, $x = -3$, $(-3,0)$

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$X = \frac{x_1 + x_2}{2}$

$X = \frac{8-3}{2}$

$\boxed{X = 5/2}$

∴ the zeros are at (8,0) and (−3,0), and the equation of the axis of symmetry is "X = 5/2"

   c. Find the coordinates of the vertex. (2 marks)

* Sub A of S into equation *

$y = x^2 - 5x - 24$

$y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) - 24$

$y = \left(\frac{25}{4}\right) - \frac{25}{2} - 24$

2 $y = \frac{25}{4} - \frac{50}{4} - \frac{96}{4}$

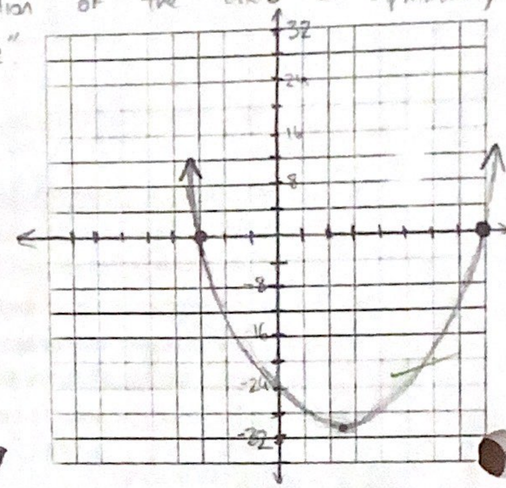
$\boxed{y = \frac{-121}{4}}$

∴ Coordinates of Vertex:

• $\left(\frac{5}{2}, \frac{-121}{4}\right)$

• $(2.5, -30.25)$

   d. Sketch the graph. (2 marks)



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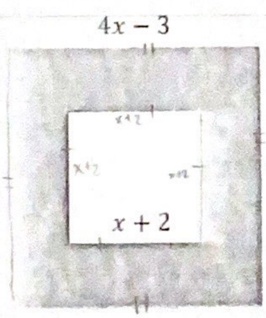
2. Find an expression for the shaded area in its simplest factored form. HINT: DO NOT EXPAND!!
(4 marks)



difference of squares

• Let x rep. (4x−3)
• Let y rep. (x+2)

$A_{\square} = (4x-3)^2 - (x+2)^2$

$A_{\square} = x^2 - y^2$ $\longrightarrow$ substitute binomials w/ variables temporarily

$A_{\square} = (x-y)(x+y)$

$A_{\square} = [(4x-3) - (x+2)][(4x-3) + (x+2)]$

$A_{\square} = [4x - 3 - x - 2][4x - 3 + x + 2]$

$A_{\square} = (3x - 5)(5x - 1)$

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∴ Simplified expression for area of shaded region:

$A = (3x-5)(5x-1)$

CHOOSE EITHER QUESTION 4 or QUESTION 5.

3. Annabelle throws a ball from the top of a cliff. The height of the ball above the base of the cliff is approximated by the model $y = -5t^2 + 10t + 75$, where y is the height in meters and t is the time in seconds.

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   a. How long does it take for the ball to reach its maximum height? (4 marks) A of S
   b. What is the maximum height of the ball? (1 mark) Y- of vertex

4. A company's profit is modelled by the relation $P = -4x^2 + 13x - 9$, where x is the number of products sold in hundreds, and P is profit in millions of dollars.
   a. What are the break-even points for the company? (3 marks)
   b. What is the maximum profit that the company can earn? (2 marks)

3) a) Factor

$y = -5t^2 + 10t + 75$

$y = -5(t^2 - 2t - 15)$

$y = -5(t^2 - 5t + 3t - 15)$

$y = -5(t - 5)(t + 3)$

∴ it takes the ball 1 second to reach its max height.

Zeros → ① $t - 5 = 0$, $t = 5$, $(5, 0)$ ② $t + 3 = 0$, $t = -3$, $(-3, 0)$

A of S

$X = \frac{x_1 + x_2}{2}$

$X = \frac{5 - 3}{2}$

$\boxed{X = 1}$

b) *sub A of S into equation*

$y = -5(1)^2 + 10(1) + 75$

$y = -5(1) + 10 + 75$

$y = -5 + 10 + 75$

$\boxed{y = 80}$

∴ the maximum height of the ball is 80 m.

5. Factor $2(a+1)^2 - (a+1) - 6$. Remember, do what you know! (3 marks)

$2(a+1)^2 - (a+1) - 6$ Let x rep. $(a+1)$

$ac = -12$
$b = -1$

$= 2x^2 - x - 6$

$= 2x^2 - 4x + 3x - 6$

$= 2x(x-2) + 3(x-2)$

$= (2x+3)(x-2)$ → sub $(a+1)$ back in

$= [2(a+1)+3][(a+1)-2]$

3

$= (2a+2+3)(a+1-2)$

$= (2a+5)(a-1)$

Part 3: Explaining Factoring (10 marks)

1. A friend in Ms. Kerr's class was absent for the most important lesson of the year (factoring by decomposition). They have asked you to help them to understand the process, so write a brief, point form note explaining how to factor a trinomial using decomposition. Include an example. (4 marks)

Standard form

$y = ax^2 + bx + c$

a, b, and c are replaced by actual numbers

$\boxed{\begin{array}{l}(?)(?) = ac \\ (?)+(?) = b\end{array}}$

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- Decomposition is a process used to factor a trinomial and put it in factored form (from standard form). The process includes many steps...
  1) With your trinomial in standard form, you want to find two numbers that ① add up to "b" and ② have the same product as "ac".
  2) Break up your "b" term into those numbers.
  3) Factor by grouping as you now have 4 terms.

*before starting, always look to common factor.

Ex $7a^2 + 19a - 6$
(a) (b) (c)

ac = -42
b = 19

① $(21)+(-2) = 19$
$(21)(-2) = -42$

∴ my numbers are 21 and -2.

② $= 7a^2 + 21a - 2a - 6$

③ $= 7a(a+3) - 2(a+3)$

$= (7a-2)(a+3)$

∴ the factored form of the trinomial is $(7a-2)(a+3)$.

2. The same friend is also having difficulty understanding how decomposition relates to expanding and simplifying from chapter 3. Use an example (it can be the one that you used in the previous question if you'd like) to show them that decomposition is just the reverse operation to expanding and simplifying two binomials. (3 marks)

- Decomposition is the exact opposite of expanding/simplifying
  - Decomposition → standard form to factored form
  - Expanding/Simplifying → factored form to standard form

$a(b+c) \xrightarrow{\text{expand/simplify}} ab + ac$
$a(b+c) \xleftarrow{\text{decomposition}} ab + ac$

Ex $7a^2 + 19a - 6$

ac = -42
b = 19

$= 7a^2 + 21a - 2a - 6$

$= 7a(a+3) - 2(a+3)$

$= (7a-2)(a+3)$

Decomposition

2.5

$(7a-2)(a+3)$

Show $7a(a+3) - 2(a+3)$

$= 7a^2 + 21a - 2a - 6$

$= 7a^2 + 19a - 6$

Expanding/Simplifying

As you can see, when we expanded/simplified our factored form, we got back to our original trinomial. This shows how the two processes are exact opposites.

3. Choose one of the shortcuts that we discussed (a = 1, b = 0, or perfect squares) and explain what the shortcut is and why it works. (3 marks)

In the "a=1" shortcut, the first term in each bracket simply becomes "x" (or whatever variable is in the question). The second terms become each of the numbers that you broke your "b" term up into through decomposition.

Ex $x^2 + 8x + 12$

ac = 12
b = 8

$= x^2 + 2x + 6x + 12$

$= (x+2)(x+6)$

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Why It Works:

The first term in each bracket is always "x" (or any other variable) because multiplying the "x"s in each bracket together is the only way to obtain the "x^2" that the rule is all about (a=1).

The numbers you found in decomposition become the second terms in the brackets because those numbers are the only ones that both add up to "b" and will give you the value of c (as a=1, "ac" will have the same value as "c").