Parent Signature:

Chapter 3: Graphs of Quadratic Relations

QR

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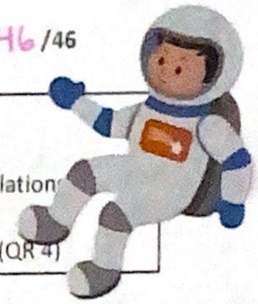


Learning Goals:
I will be able to:

- Explain properties of quadratic relationships, and relate the various forms of quadratic relation
- Solve quadratic equations and interpret their solutions; (QR 3)
- Apply my understanding of quadratic relations to a variety of problem solving situations. (QR 4)

Instructions and Hints:

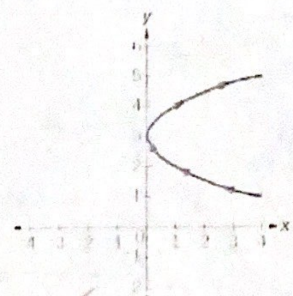
- Read all questions carefully and be sure to answer all parts! **Don't lose marks for not reading questions.**
- Be careful with your signs when you are finding zeros and writing equations of parabolas.
- Make sure that all answers are organized and legible. If I can't follow or read it I am not marking it.
- Remember that you need to know **three points** to sketch a graph!

1. State whether each relationship given below is linear, quadratic, or neither. (2 marks)

a)

Quadratic

b)



Neither

c) $y = -2x^2 - 5x + 6$

Quadratic

d)

x	y
-2	¼
-1	½
0	1
1	2
2	4

1/4, 1/2, 1, 2

1/4, 1/2, 1

Neither

2

2. Briefly explain your reasoning for each part of question 1 (in the space below, not more than that!) (2 marks)

a) Symmetric ~~U~~-shape that opens upwards (parabola) → ∴ quadratic

b) Parabolic shape but it opens sideways → ∴ neither

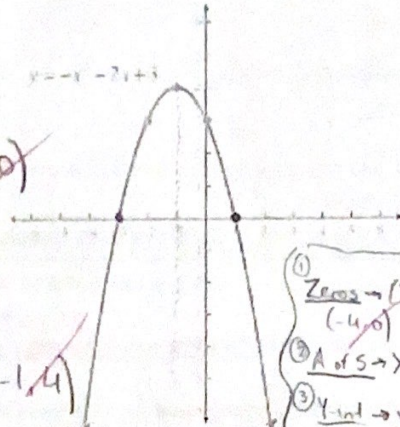
c) Equation is 2^{nd} degree → ∴ Quadratic

2

d) Neither the first, nor the second differences are equal → ∴ neither

3. Given the following quadratic relations, state the zeros, the y – intercept, the equation of the axis of symmetry, and the coordinates of the vertex. (6 marks)

a.



Zeros → (-3,0), (1,0)

Y-Int → (0,3)

A of S → X = -1

Coordinates of Vertex → (-1,4)

6

b. $y = -\frac{1}{3}(x-3)(x+4)$

① $x-3=0$ $\boxed{x=3}$ $x+4=0$ $\boxed{x=-4}$

② $X = \frac{x_1+x_2}{2}$

$X = \frac{3-4}{2}$

$\boxed{X = \frac{-1}{2}}$

③ $y = \frac{-1}{3}(0-3)(0+4)$

$y = \frac{-1}{3}(-3)(4)$

$y = \frac{12}{3}$

$\boxed{y=4}$

④ $y = \frac{-1}{3}\left(\frac{-1}{2} \ldots\right.$

$y = \frac{-1}{3}\left(\frac{-1}{2} \ldots\right.$

$y = \frac{-1}{3}\left(\frac{-7}{2}\right)\left(\frac{7}{2}\right)$

$y = \frac{-1}{3}\left(\frac{-49}{4}\right)$

$\boxed{y = \frac{49}{12}}$

① Zeros → (3,0), (-4,0)

② A of S → $X = \frac{-1}{2}$

③ Y-int → y = 4

④ Vertex $\left(\frac{-1}{2}, \frac{49}{12}\right)$

4. A quadratic relation is modelled by $y = -2(x-3)(x+1)$. Accurately sketch the graph of the relation. (4 marks)

Zeros:

① $x-3=0$ → (3,0)

$\boxed{x=3}$

② $x+1=0$

$\boxed{x=-1}$ → (-1,0)

A of S:

$X = \frac{-1+3}{2}$

$\boxed{X=1}$

Vertex:

$y = -2(1-3)(1+1)$

$y = -2(-2)(2)$

$\boxed{y=8}$

$\boxed{\therefore \text{Vertex} \rightarrow (1,8)}$

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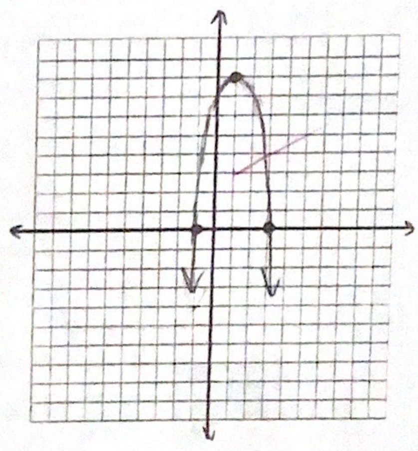
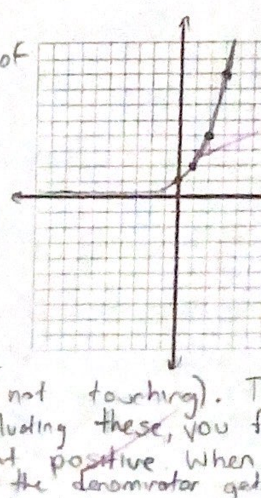


5. Draw the graph of y = 2^x. State the y – intercept and explain why y = 2^x does not have any x-intercepts. (3 marks)

The y-intercept is 1 as anything to the exponent of 0 evaluates to 1.



$y=2^x$ has no x-intercepts because the equation will always have a positive value. You can see how the line accelerates in the positive direction, ∴ that will continue its trend and not cross the x-axis. However, the line is very close to the x-axis on the ⊖ side of the x-axis (but not touching). This is because this part represents results of negative exponents. When evaluating these, you find the reciprocal of the base, and then make the exponent positive. When finding a reciprocal, the numerator will always be 1 or greater. Only the denominator gets bigger, makes the fraction smaller. Since the numerator will always have some ⊕ value, it never crosses the x-axis.

3

6. What is the value of a base raised to the exponent zero? Solve $(-\frac{1}{2})^0$ to support your answer. (1 mark)

1 $\quad \left(-\frac{1}{2}\right)^0 = 1 \quad \therefore$ any based raised to ~~the~~ the exponent 0 evaluates to 1.

7. What effect does a negative exponent have on its base? Solve $(4)^{-3}$ to support your answer. Be sure to report your answer as a fraction, not a decimal. (2 marks)

To solve a negative exponent, you (1) flip the base (reciprocal) and (2) change the exponents sign (negative to positive).

2 $\quad (4)^{-3} = \left(\frac{1}{4}\right)^3 = \left(\frac{1^3}{4^3}\right) = \frac{1}{64} \longrightarrow 4^{-3} = \frac{1}{64}$

8. Expand and simplify each of the following expressions. (QR3 8 marks)

a. $(2x + 3)(5x - 1)$

$= 10x^2 - 2x + 15x - 3$

$= 10x^2 + 13x - 3$

b. $(2x - 5)^2$

$= (2x-5)(2x-5)$

$= 4x^2 - 10x - 10x + 25$

$= 4x^2 - 20x + 25$

c. $-\frac{2}{3}(x - 1)(2x - 9)$

$= \frac{-2}{3}(2x^2 - 9x - 2x + 9)$

8 $\quad = \frac{-2}{3}(2x^2 - 11x + 9)$

$= \frac{-4}{3}x^2 + \frac{22}{3}x - \frac{18}{3}$

$= \frac{-4}{3}x^2 + \frac{22}{3}x - 6$

9. A student solution is provided below. You need to identify their error, explain it (in words) and offer advice to help the student learn from this error. (3 marks)

Question: Write $y = 3(x - 5)(x + 1)$ in standard form.

Solution: $y = 3(x-5)(x+1)$

$= (3x-15)(3x+3)$

$= 3x(3x+3) - 15(3x+3)$

$= 9x^2 + 9x - 45x - 45$

3 $\quad y = 9x^2 - 36x - 45$

Explanation of Error:

The student distributed the 3 into both brackets at once, when you should only multiply two polynomials at once.

Advice:

Next time, the student should only multiply 2 polynomials at a time. For example, they could have multiplied "(x-5)" and "(x+1)", then multiplied its result by the outer 3. Distributing the 3 into both brackets is not possible as there's only one 3 (distributing into both is like "duplicating" it).

10. Write an equation, **in standard form**, for a parabola that has zeros at (-2, 0) and (5, 0) and a maximum value of 4. (5 marks)

$r = -2$
$s = 5$
$x = \frac{3}{2}$
$y = 4$

$X = \frac{r+s}{2}$
$X = \frac{-2+5}{2}$
$X = \frac{3}{2}$

5

$y = a(x-r)(x-s)$
$4 = a\left(\frac{3}{2}+2\right)\left(\frac{3}{2}-5\right)$
$4 = a\left(\frac{3}{2}+\frac{4}{2}\right)\left(\frac{3}{2}-\frac{10}{2}\right)$
$4 = a\left(\frac{7}{2}\right)\left(\frac{-7}{2}\right)$
$4 = a\left(\frac{-49}{4}\right)$

$\frac{4}{-49/4} = \frac{-\frac{49}{4}a}{-49/4}$
$\frac{4}{1}\left(\frac{-4}{49}\right) = a$
$\boxed{\frac{-16}{49} = a}$

(factored form)
$\therefore y = \frac{-16}{49}(x+2)(x-5)$
$y = \frac{-16}{49}(x^2 - 5x + 2x - 10)$
$y = \frac{-16}{49}(x^2 - 3x - 10)$
$\boxed{y = \frac{-16}{49}x^2 + \frac{48}{49}x + \frac{160}{49}}$
(standard form)

$\therefore$ the equation is "$y = \frac{-16}{49}x^2 + \frac{48}{49}x + \frac{160}{49}$".

11. A parabolic stone archway to be built over a street in downtown Windsor needs to be 24 m wide and have a maximum height of 9 m. Determine an equation, in factored form, that models the archway. (4 marks)

Zeros → (0, 0), (24, 0)
(24 m wide)

$X = \frac{0+24}{2}$
$\boxed{X = 12}$ → (a of s)

Vertex → (12, 9)
A of S, max. height

4

$r = 0$
$s = 24$
$x = 12$
$y = 9$

$y = a(x-r)(x-s)$
$9 = a(12-0)(12-24)$
$9 = a(12)(-12)$
$9 = -144a$

$\frac{9}{-144} = \frac{-144a}{-144}$
$\boxed{\frac{-9}{144} = a}$

Equation:
$y = \frac{-9}{144}(x-0)(x-24)$
$y = \frac{-9}{144}(x)(x-24)$

Choose ONE of the next two questions and provide a complete solution. (6 marks)

12. The population of a city is modelled by $P = -0.5(t-30)(t+10)$, where P is the population in thousands and t is the time in years.
   a. How many years will it take for the population to reach zero?
   b. What is the current population of the city?
   c. When will the population reach a maximum?
   d. What is the maximum population?

13. A baseball is thrown from the top of a building. Its path is modelled by $y = -5(x-6)(x+5)$, where x is time is seconds from when the ball is thrown and y is the height of the ball in meters.
   a. How tall is the building?
   b. When does the ball hit the ground?
   c. When does the ball reach its maximum height?
   d. What is the maximum height of the ball?

6

a) *y-int needed* (when x = 0)
$y = -5(0-6)(0+5)$
$y = -5(-6)(5)$
$\boxed{y = 150}$
$\therefore$ the building is 150 m tall.

b) Zeros needed *
$y = -5(x-6)(x+5)$
$x - 6 = 0$, $x = 6$
$x + 5 = 0$, $x = -5$
$\therefore$ the ball hits the ground after 6 seconds.
(not possible for ball to be thrown at -5 seconds → theoretical)

c) *A of S needed* (says "when")
$X = \frac{r+s}{2}$ (r, s are zeros)
$X = \frac{6-5}{2}$
$\boxed{X = \frac{1}{2}}$
$\therefore$ the ball reaches its maximum height after half (0.5 seconds) of a second.

d) *max. height needed*
*sub "A of S" into equation
$y = -5(x-6)(x+5)$
$y = -5\left(\frac{1}{2}-6\right)\left(\frac{1}{2}+5\right)$
$y = -5\left(\frac{1}{2}-\frac{12}{2}\right)\left(\frac{1}{2}+\frac{10}{2}\right)$
$y = -5\left(\frac{-11}{2}\right)\left(\frac{11}{2}\right)$
$y = -5\left(\frac{-121}{4}\right)$
$y = \frac{605}{4}$
$\boxed{y = 151.25 \text{ m}}$
$\therefore$ the max height of the ball is 151.25 m.