Parent Signature:

Test
Chapter 6: Quadratic Equations

QR

41.5/42 (99%)

Learning Goals:
- I will be able to solve quadratic equations and interpret their solutions (QR3)
- I will be able to apply my understanding of quadratic relations to a variety of pro… solving situations (QR4)

Instructions and Hints:

- Please read all questions carefully and provide the required information in your solution.
- Define your variables if you are creating your own equations, and include a concluding statement with all word problems.
- Extra time will not be provided, please make sure that you choose the most efficient way to determine your answer! You have 80 minutes ☺
- Check the reasonableness of all solutions!
- Follow instructions and pay attention to choice!

1. Find the roots of each of the following quadratic equations **using the most appropriate method**.

a. $x^2 - 4x - 10 = 2$ (3 marks)

$x^2 - 4x - 10 - 2 = 0$
$x^2 - 4x - 12 = 0$
$x^2 - 6x + 2x - 12 = 0$
$(x-6)(x+2) = 0$
① $x - 6 = 0$, $x = 6$
② $x + 2 = 0$, $x = -2$
ac = -12, b = -4
3

b. $-3(x+2)^2 + 48 = 0$ (4 marks)

$-3(x+2)^2 = -48$
$\sqrt{(x+2)^2} = \sqrt{16}$
$x + 2 = \pm 4$
① $x + 2 = 4$, $x = 4 - 2$, $x = 2$
② $x + 2 = -4$, $x = -4 - 2$, $x = -6$
4

c. $3x(3x-1) + 2x = x^2 - 4(x-3)$ (5 marks)

$9x^2 - 3x + 2x = x^2 - 4x + 12$
$9x^2 - x = x^2 - 4x + 12$
$9x^2 - x^2 - x + 4x - 12 = 0$
$8x^2 + 3x - 12 = 0$ → use quadratic formula
$a = 8$, $b = 3$, $c = -12$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-3 \pm \sqrt{(3)^2 - 4(8)(-12)}}{2(8)}$
$x = \frac{-3 \pm \sqrt{9 + 384}}{16}$
$x = \frac{-3 \pm \sqrt{393}}{16}$
① $x = \frac{-3 + \sqrt{393}}{16}$, $x = 1.05$
② $x = \frac{-3 - \sqrt{393}}{16}$, $x = -1.43$
5

a b c

2. Determine the **number of real roots** that the parabola represented by $y = 8x^2 - 3x + 4$ would have without solving. Show your work and justify your conclusion. (3 marks)

use discriminant to determine # of real roots (4)

$a = 8$, $b = -3$, $c = 4$

$D = b^2 - 4ac$
$D = (-3)^2 - 4(8)(4)$
$D = 9 - 128$
$D = -119$

3

the discriminant is negative, meaning it cannot be square rooted. ∴ there are no real roots.

Choose TWO of the next THREE problems to solve. (5 marks each)

3. A stunt diver is diving off of a platform that is 12 m high into a pool of water that is 45 cm deep. The height, h, in meters of the stunt diver above the water is modelled by $h = -4.9t^2 + 1.2t + 12$, where t is time in seconds after starting the dive.
   a. When does the stunt diver reach their maximum height? (2 decimal places) k (vertex)
   b. When does the diver reach a height of 9 m? (2 decimal places) (root)
4. The cost, C, in dollars to hire landscapers to weed and seed a local park can be modelled by $C = 6x^2 - 60x + 900$, where x is the number of landscapers hired to do the work.
   a. How many landscapers should be hired to minimize the cost? vertex (k)
   b. How many landscapers can the park afford to hire if they have $5450 available to spend? roots
5. Claire sells specialty teddy bears at various summer festivals. Her profit for a week, P, in dollars is modelled by $P = -0.1n^2 + 30n - 1200$, where n is number of teddy bears sold during the week.
   a. How many teddy bears does she need to sell to maximize her profit?
   b. How many teddy bears does she need to sell if she needs to make at least $500 profit?

3) a) $X = \frac{-b}{2a}$ *find h of s*

$a = -4.9$, $b = 1.2$, $c = 12$

$X = \frac{-1.2}{2(-4.9)}$

$X = \frac{-1.2}{-9.8}$

$X = 0.12$

∴ the stunt diver reaches their maximum height after 0.12 seconds.

b) $h = -4.9t^2 + 1.2t + 12$
$9 = -4.9t^2 + 1.2t + 12$ (-9)
$0 = -4.9t^2 + 1.2t + 3$

$a = -4.9$, $b = 1.2$, $c = 3$

$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$X = \frac{-1.2 \pm \sqrt{(1.2)^2 - 4(-4.9)(3)}}{2(-4.9)}$

$X = \frac{-1.2 \pm \sqrt{60.24}}{-9.8}$

① $X = \frac{-1.2 + \sqrt{60.24}}{-9.8}$

$X = -0.67$

② $X = \frac{-1.2 - \sqrt{60.24}}{-9.8}$

$X = 0.91$

use this value (① time doesn't exist)

∴ the stunt diver reaches a height of 9m after 0.91 seconds.

4) a) $X = \frac{-b}{2a}$ *find h of s*

$a = 6$, $b = -60$, $c = 900$

$X = \frac{60}{2(6)}$

$X = \frac{60}{12}$

$X = 5$

∴ 5 landscapers should be hired to minimize costs.

b) $C = 6x^2 - 60x + 900$
$5450 = 6x^2 - 60x + 900$ → solve for # of workers
$0 = 6x^2 - 60x + 900 - 5450$
$0 = 6x^2 - 60x - 4550$ → plug into formula

$a = 6$, $b = -60$, $c = -4550$

$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$X = \frac{60 \pm \sqrt{(-60)^2 - 4(6)(-4550)}}{2(6)}$

$X = \frac{60 \pm \sqrt{112800}}{12}$

① $X = \frac{60 + \sqrt{112800}}{12}$

$X = 32.99$
(33)

② $X = \frac{60 - \sqrt{112800}}{12}$

$X = -22.99$
(cannot have (-) workers)

∴ the park can afford to hire 33 workers.

5

5

6. Below you will find a test question, as well as a student's solution. Your job is to identify their error(s) and describe it, and then explain a better strategy to solve so that they can learn from their mistake. (3 marks)

Question: Determine the roots of $2(x+4)^2 - 6 = 2$.

Solution:

$2(x^2+16) - 6 = 2$

$2x^2 + 32 - 6 = 2$

$2x^2 + 26 = 2$

$2x^2 = -24$

$x^2 = -12$

$x^2 = \pm\sqrt{-12}$

not possible, so no real roots

2.5

Description of Error: The student squared each term in $x+4$ individually and didn't use distributive property (or FOIL). This is why there is no "bx" term. They also tried to put it in standard form to rearrange, though solving by rearranging should only be done in vertex form.

Improved Strategy: Next time, when squaring a binomial, they should write it out twice (as two separate brackets) to understand that you must use distributive property. Also, if they're in standard form, they should only either use factoring or the quadratic formula to solve (rearranging is for vertex form).

CHOOSE ONE OF THE NEXT TWO QUESTIONS TO ANSWER!! (3 marks)

7. What is the discriminant, and what does it tell you about a quadratic relationship?

8. Explain two ways to find the roots of a quadratic equation in standard form.

① Finding roots by factoring: In this method, you start by moving all terms to one side (if necessary). Then, you have to factor (by decomposition, grouping, shortcut, etc.). Then, set each factor/bracket to 0 and solve for each x-value. You should be left with 2 x values.

Ex: $x^2 + 5x + 3 = -1$

$x^2 + 5x + 4 = 0$

$x^2 + 1x + 4x + 4 = 0$

$(x+1)(x+4) = 0$

$x+1=0$ → $x=-1$; $x+4=0$ → $x=-4$

3

② Quadratic Formula: When factoring isn't possible (ex: b/c of decimals), you use the quadratic formula. You must first rearrange to put all terms to one side, then you use your a, b, and c values in the formula.

Formula → $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

(*) when you square root, you must put "±" in front and solve in both cases (as the root can be either positive or negative).

9. Algebraically determine the points of intersection of the parabolas $y = 2x^2 + 5x - 12$ and $y = x^2 + 6x - 6$. Use what you know (you may need to recall some information from unit 1!). (5 marks)

5

* substitution

$y = 2x^2 + 5x - 12$

$y = x^2 + 6x - 6$

$2x^2 + 5x - 12 = x^2 + 6x - 6$

$2x^2 - x^2 + 5x - 6x - 12 + 6 = 0$

$x^2 - x - 6 = 0$

$ac = -6$, $b = -1$

$x^2 - 3x + 2x - 6 = 0$

$(x-3)(x+2) = 0$

① $x - 3 = 0$ → $x = 3$

② $x + 2 = 0$ → $x = -2$ ← sub these back into the equation.

① $y = 2(3)^2 + 5(3) - 12$

$y = 2(9) + 15 - 12$

$y = 18 + 15 - 12$

$y = 21$ POI #1: $(3, 21)$

② $y = 2(-2)^2 + 5(-2) - 12$

$y = 2(4) - 10 - 12$

$y = 8 - 10 - 12$

$y = -14$ POI #2: $(-2, -14)$

∴ the points of intersection of the two parabolas are $(3, 21)$ and $(-2, -14)$.

CHOOSE ONE OF THE FOLLOWING PROBLEMS AND PROVIDE A FULL SOLUTION. Remember that you MUST define your variables, create a quadratic expression, and use it to solve. Write a concluding statement when you are done that answers the question! (6 marks)

$R = (5+0.5x)(320-20x)$

$A_T =$

10. Tickets to a school dance cost $5, and the projected attendance is 320 people. For every $0.50 increase in price, the dance committee projects that attendance will decrease by 20. Create a quadratic relation to model the situation. Use your relation to determine the ticket price(s) will generate $1620 in revenue.

11. Delaney has a garden that measures 20 m by 25 m. She wants to create a walkway of uniform width around the garden. She wants the area of the walkway to be equal to the area of the garden. Create a quadratic relation to model this situation. Use your relationship to determine the width of the walkway to the nearest tenth of a meter.

10) ① Let R represent total Revenue
② Let x represent the # of price changes

$R = (\text{price})(\text{quantity})$
$R = (5+0.5x)(320-20x)$

$1620 = (5+0.5x)(320-20x)$
$1620 = 1600 - 100x + 160x - 10x^2$
$1620 = -10x^2 + 60x + 1600$
$0 = -10x^2 + 60x - 20$
$0 = -x^2 + 6x - 2$

Use quadratic formula → $a = -1$, $b = 6$, $c = 2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-2)}}{2(-1)}$
$x = \frac{-6 \pm \sqrt{28}}{-2}$

① $x = \frac{-6 + \sqrt{28}}{-2}$
$x = 0.35$

② $x = \frac{-6 - \sqrt{28}}{-2}$
$x = 5.65$

Ticket Prices: ① $5 + 0.5(0.35) = 5.175 = \$5.18$ ② $5 + 0.5(5.65) = 7.825 = \$7.83$

∴ ticket prices of $5.18 and $7.83 will generate $1620 in revenue.