E 4600
ll 2018

91/105

I certify that the work being submitted is my own, and that no unauthorized assistance was obtained.

Name (please print):

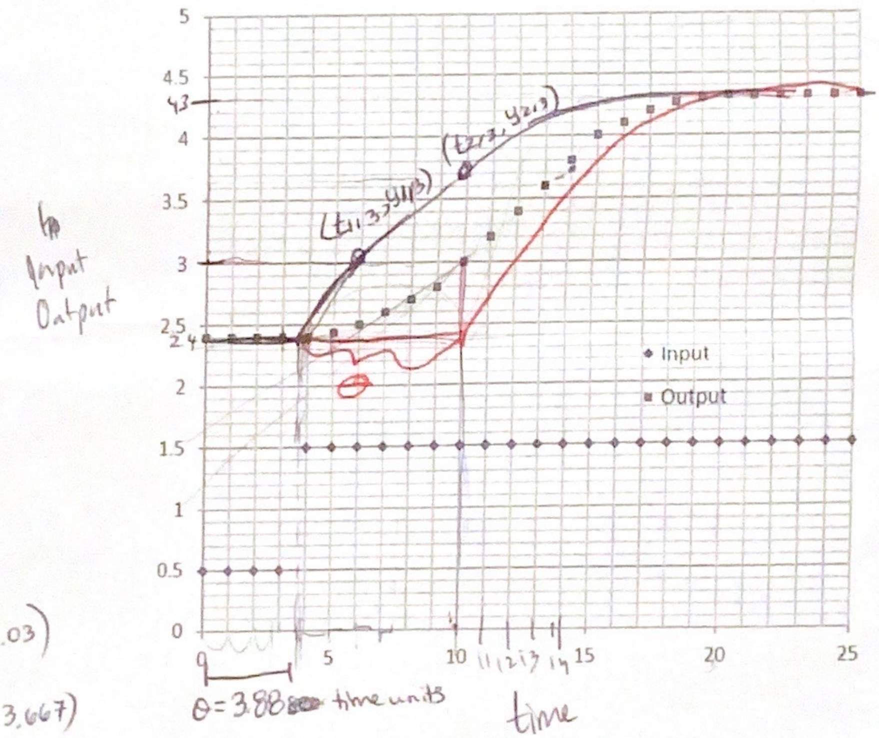
Signature:

Exam #2

(Closed book, closed notes)

1. For the experimental data below:
   (a) Develop a first order plus dead time (FOPDT) model for the input-output system shown below. (10 pts) 10
   (b) Sketch your model prediction of the output on the axes by showing the main model parameters on the graph. (10 pts) 6
   (c) Can you determine the order of the process system generating this output? Explain your answer. (5 pts) 0

16



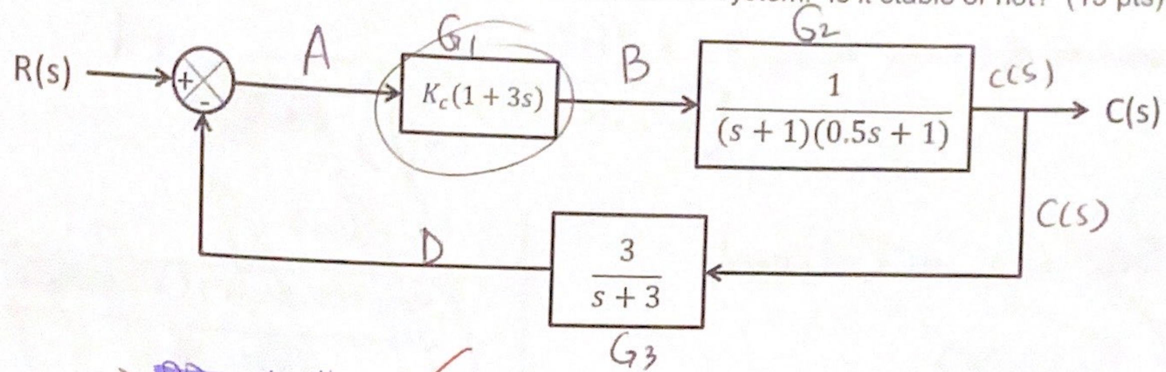
$(t_{1/3}, y_{1/3}) = (6, 3.03)$

$(t_{2/3}, y_{2/3}) = (9.7, 3.667)$

$\theta = 3.88$ ~~seconds~~ time units (deadtime)

The block diagram below illustrates a feedback control system for a controlled variable C(s)
(a) What kind of controller or control algorithm is being used in this system? (5 pts) 5
(b) Develop the overall transfer function C(s)/R(s). (5 pts) 5
(c) Determine the final value of C(t) if K_c is 9 and R(t) exhibits a step change of +12 units. (10 pts) 10
(d) Evaluate the stability of this process feedback control system. Is it stable or not? (15 pts) 15

35



a) PD controller ✓

P → K_c

D → $T_D s$ term

There is no $\frac{1}{s}$ term, so no I controller

b) $\frac{C(s)}{R(s)} =$

$$\frac{C(s)}{R(s)} = \frac{\text{multiply straight through } R(s) \text{ to } C(s)}{\text{multiply loop} + 1} = \frac{G_1G_2}{G_1G_2G_3+1}$$ ✓

$$\Rightarrow \frac{K_c(1+3s)\left(\frac{1}{(s+1)(0.5s+1)}\right)}{(K_c(1+3s))\left(\frac{1}{(s+1)(0.5s+1)}\right)\left(\frac{3}{s+3}\right)+1}$$ ✓

c) $K_c = 9$

$R(t) = 12$

$R(s) = \frac{12}{s}$ ✓

FVT → $\lim_{s\to 0} [s(F(s))] = \lim_{t\to\infty} [f(t)]$

$$F(s) \Rightarrow \frac{G_1G_2}{G_1G_2G_3+1} * R(s) \frac{12}{s}$$

$$\lim_{s\to 0}\left(\cancel{s} \cdot \frac{\frac{K_c(1+3s)}{(s+1)(0.5s+1)}}{K_c(1+3s)\left(\frac{1}{(s+1)(0.5s+1)}\right)\left(\frac{3}{s+3}\right)+1}\right) \cdot \frac{12}{\cancel{s}} \Rightarrow \frac{\frac{9(1)}{(1)(1)}}{9(1)\left(\frac{1}{1}\right)\left(\frac{3}{3}\right)+1}$$

$$= \frac{9 * 12}{9+1} \Rightarrow \boxed{10.8}$$ ✓

d) Stability

$\frac{C(s)}{R(s)} \Rightarrow$ take denominator to assess Stability with Routh Array ~~and~~

Denominator of transfer function: $\frac{K_c(1+3s)}{\left[(s+1)(0.5s+1)\left(\frac{3}{s+3}\right)\right]} + 1$

Set $= 0$ and expand

$$\frac{K_c(1+3s)}{(s+1)(0.5s+1)\left(\frac{3}{s+3}\right)} + 1 = 0$$

$$\frac{K_c + 3K_c s}{\frac{3(0.5s^2) + 3(s) + 3(0.5s) + 3}{s+3}} + 1 \cdot \frac{s+3}{s+3} = 0$$

$$\frac{9K_c + 9K_c s}{\frac{1.5s^2 + 3s + 1.5s + 3 + s + 3}{s+3}} = 0$$

$$\frac{9 + 27s}{3(1.5s^3 + 3s^2 + 1.5s^2 + 3(3s) + 3s^2 + 3(3s))} = 0$$

$$\frac{9 + 27s}{4.5s^3 + 9s^2 + 4.5s^2 + 9s + 3s^2 + 9s}$$

16.5 18s

$$\frac{K_c(1+3s)}{(s+1)(0.5s+1)(3)} \cdot \frac{1}{s+3} = -1$$

$$s+3 = \frac{K_c(1+3s)}{(s+1)(0.5s+1)(3)}$$

$$\frac{K_c(1+3s)(3)}{(s+3)(0.5s+1)(s+1)} = -1$$ ✓

$$\frac{3K_c + 9K_c s}{0.5s+1} = -s^2 - 4s - 3$$

$$3K_c + 9(K_c)s = 0.5s^3 - 2s^2 - 1.5s - s^2 - 4s - 3$$

$$27 + 81s = 0.5s^3 + 3s^2 + 5.5s + 3$$

① $0.5s^3 + 3s^2 + 86.5s + 30 = 0$ ✓

criteria #1: all coefficients are positive → satisfies first criteria for stability ✓

② Routh array:

$a_3 = 0.5$, $a_1 = 86.5$, $a_2 = 3$, $a_0 = 30$

$$\begin{bmatrix} a_3\ 0.5 & 86.5\ a_1 & 0 \\ a_2\ 3 & 30\ a_0 & 0 \\ \frac{a_2 a_1 - a_3 a_0}{a_2} & & 0 \\ a_0\ 30 & & 0 \end{bmatrix}$$

$$\frac{(3)(86.5) - (0.5)(30)}{3} = 81.5$$

criteria #2: The whole first column of the array is positive so the system is stable ✓

3. The set point (SP) of the control system below is given a step change of 0.2 units.

(a) The blocks shown represent a process system and a controller. Identify as many parameters of the process system as you can and explain what type of open loop behavior you would expect the system to display for a direct step change input. (5 pts) 5

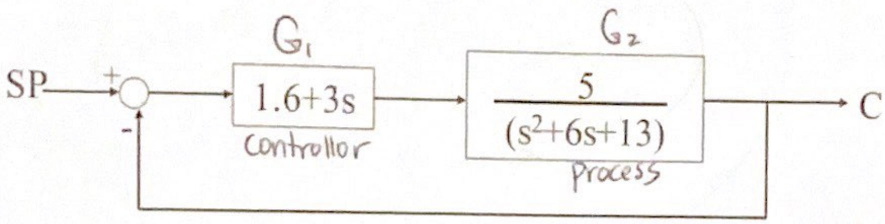
(b) Determine the final value of C after the setpoint change. (10 pts) 10

(c) Describe the closed loop dynamic behavior of C after the set point change. (15 pts) 10

(d) Calculate the offset in C after the setpoint change. (15 pts) 0

40

(e) If you wished to sabotage the operation of this system by altering the controller settings to produce unstable behavior after a 0.2 unit setpoint change, how would you do it? Describe with supporting numbers. (15 pts) 15



$$\text{Transfer function} = \frac{\text{output}}{\text{input}} = \frac{C(s)}{SP(s)} = \frac{G_1G_2}{G_1G_2+1}$$

a) $1.6 + 3s$ is the controller

This does not have $\frac{1}{s}$ ~~stake~~ ~~alone~~ terms, so it is a PD controllor

$K_c(1+\tau_D s) \Rightarrow 1.6 + 3s$

~~K_c~~ $K_c = 1.6$, $\tau_D = 1.875$ Parameters for G_1 (controller)

$1.6(1 + 1.875s)$

The process is a second order process ~~and~~ because of the order of s in the denominator $\rightarrow \frac{5}{s^2+6s+13} \rightarrow \frac{5/13}{\frac{s^2}{13}+\frac{6s}{13}+1}$

a) process parameters

~~$C(s) = G_1G_2$~~

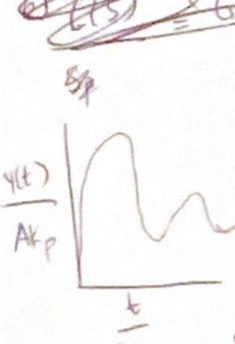
S/P

$K_p = 5/13$ Parameters for G_2

$\tau_p^2 s^2 = \frac{1}{13}s^2$

$\tau_p^2 = \frac{1}{13}$

$\tau_p = \sqrt{1/13}$



$2\zeta\tau_p s = \frac{6}{13}s$

$2\zeta\left(\sqrt{1/13}\right) = \frac{6}{13}$

$\zeta < 1$ → the open loop behavior of the process will be underdamped. There will be exponentially ~~increasing~~ oscillations

decreasing → $\zeta \cong 0.83$

1) $G_p(s) = \dfrac{K_p e^{-\theta_p s}}{\tau_p s+1}$

Neha

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$$\tau_p = \frac{t_{2/3} - t_{1/3}}{0.7}$$

$$\theta_p = t_{1/3} - 0.4\tau_p$$

$$K_p = \frac{\Delta y}{\Delta u} = \frac{y(25) - y(0)}{u(25) - u(0)} = \frac{4.3 - 2.4}{1.5 - 0.5} = 1.9$$

~~4.3-2.4~~

$$y_{2/3} = \tfrac{2}{3}\Delta y + 2.4 = 3.6667$$

$$y_{1/3} = \tfrac{1}{3}\Delta y + 2.4 = 3.03$$

$$\Delta y = 4.3 - 2.4 = 1.9$$

$t_{1/3} = 10$ (time @ $y \approx 3$) $- 4$ (time where step function began) $= 6$

$t_{2/3} = 13.7$ (time @ $y \approx 3.67$) $- 4$ $= 9.7$

$$\tau_p = \frac{t_{2/3} - t_{1/3}}{0.7} = \frac{9.7 - 6}{0.7} = 5.2857$$

$$\theta_p = t_{1/3} - 0.4\tau_p = 6 - 0.4(5.28) = 3.88$$

a) $\therefore G_p(s) = \dfrac{K_p e^{-\theta_p s}}{\tau_p s + 1} = \boxed{\dfrac{1.9\, e^{-3.88 s}}{5.28 s + 1}}$

b) On graph (exam page)

c) ~~4th order because deadtime ≈ 4 (3.88)~~

~~4th order because of the deadtime ≈ 4~~

4th order because $\theta \approx 4$ ✗

#3b)

$$C(s) = \left(\frac{G_1 G_2}{G_1 G_2 + 1}\right)\left(S_p(s)\right)$$

$$S_p(t) = 0.2$$
$$S_p(s) = \frac{0.2}{s}$$

$$= \frac{0.2}{s}\underbrace{\left(\frac{(1.6+3s)\left(\frac{5}{s^2+6s+13}\right)}{(1.6+3s)\left(\frac{5}{s^2+6s+13}\right)+1}\right)}_{\to F(s)}$$

$$\text{FVT} \to \lim_{s\to 0} sF(s) = \lim_{t\to\infty}[f(t)]$$

$$\lim_{s\to 0}\left(\cancel{s}\cdot\frac{0.2}{\cancel{s}}\cdot F(s)\right) = 0.2(F(s))$$

$$= \left(0.2\left(\frac{(1.6+0)\left(\frac{5}{0+0+13}\right)}{(1.6+0)\left(\frac{5}{0+0+13}\right)+1}\right)\right) ✓$$

$$= 0.2\left(\frac{1.6\cdot\frac{5}{13}}{\left(1.6\cdot\frac{5}{13}\right)+1}\right) = 0.2\left(\frac{0.61538}{1.6153}\right) = \boxed{0.446152} \times 0.2$$

Final value $C(s)$

c) Closed loop dynamic behavior given by characteristic eqn of a closed loop system $\to$ $\cancel{G_1}\cdot\cancel{G_2} + 1 = 0$

$G_c \quad G_p$

$$G_c G_p = -1$$

$$\left(\frac{5}{s^2+6s+13}\right)\left(1.6+3s\right) = -1$$

$$\frac{8+15s}{s^2+6s+13} = -1$$

$$8+15s = -s^2-6s-13$$
$$-s^2-15s-6s-13-8=0$$
$$-s^2-21s-21=0$$
$$s^2+21s+21=0$$

$$-21 \pm \sqrt{(21)^2-4(1)(21)}/2(1)$$

$$\frac{-21 \pm 18.89}{2} \quad -1.055, \quad -19.945$$

The ~~roots~~ poles of the closed loop process are both negative, thus it is a stable system.

behavior ??

ξ ??

$$C(s) = \left(\frac{G_1 G_2}{G_1 G_2 + 1}\right)\left(S_p\cancel{(s)}\right) \quad \text{after}$$
$$\frac{0.2}{s}$$

Apply FVT → from b), 0.446152 is C_{final}

$$C(s) = \frac{G_1 G_2}{G_1 G_2 + 1}\left(Sp\cancel{(s)}\right) \quad \text{before} \quad (\text{no change})$$
$$1$$

Apply FVT, ~~G(s)~~

$$\lim_{s \to 0} s(F(s)) \to s\left(\frac{1.6 \cdot 5/13}{(1.6 \cdot 5/13)+1}\right)\frac{12}{s} = 0 ???$$

offset = SP − (final value)

$\boxed{\text{The offset is } 0.446152}$ ✗

3e) I would attempt to get the roots to ~~be~~ have positive real components. After the setpoint change, the closed loop characteristic eqn is applied,

So that was given in part c) as:

$G_c G_p$ ~~and~~ $+1=0$

$G_c G_p = -1$ ✓

If Kp was ~~reduced to the~~ increased to a very large number such as 20, the eqn (in standard form) would be

$$\frac{20}{\frac{s^2}{13} + \frac{6s}{13} + 1} \to Kp = 20$$

~~The d~~ The process eqn would be given as $\frac{260}{s^2+6s+13} \to \frac{\cancel{260}/13 \; 20}{\frac{s^2}{13} + \frac{6s}{13} + \frac{\cancel{13}}{\cancel{13}}1}$

Now when finding the poles, they would be positive

$$(1.6 + 3s)\left(\frac{260}{s^2+6s+13}\right) = -1 \Rightarrow \frac{260(1.6+3s)}{s^2+6s+13} = -1$$

$$\frac{416 + 780s}{s^2+6s+13} \to 416 + 780s = -s^2 - 6s - 13$$

$$-s^2 - 786s - 429 = 0$$

$$\frac{786 \pm \sqrt{786^2 - 4(-1)(-429)}}{-2}$$

There is now a positive root which can make the system unstable →

$$\frac{786 \pm \sqrt{\cancel{617796}}\; 787.09}{-2}$$

0.545 ✓

−786.545