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Differential Calculus
03-62-139/140, Fall 2015
Lab Assignment 3

[redacted]

Show ALL your work to receive FULL credit.

1. Find all values of x in the given interval that satisfy the inequality.

   (a) (7 marks) $2\sin^2 x - 1 \le 0$ for $x \in [0, 2\pi]$

   (b) (7 marks) $\tan x < 2\sin x$ for $x \in [0, \pi]$

2. Graph the functions by hand, not by plotting points, but by starting with the graph of one of the standard functions and applying the appropriate transformations.

   (a) (3 marks) $y = \dfrac{1}{2x-4}$

   (b) (4 marks) $y = 1 - 2\cos x$

   (c) (3 marks) $y = \ln(1-x)$

3. Determine if the following functions are one-to-one, even, odd, periodic. Also find the interval of increase and interval of decrease. → know tan, e^x, ln, log graphs

   (a) (5 marks) $f(x) = x^2 + 4x + 2$.

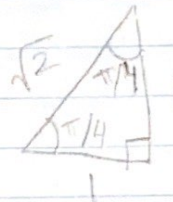
   Periodic? (b) (5 marks) $f(x) = \tan 2x$ for $x \in (-\frac{3\pi}{4}, \frac{3\pi}{4})$.

4. Find the functions $f+g$, fg, and $f \circ g$.

   (a) (3 marks) $f(x) = \dfrac{x}{2x-1}$, $g(x) = \tan x + e^x$

   (b) (3 marks) $f(x) = 5x^2 - 2x$, $g(x) = \sqrt[3]{1+x} - 2$

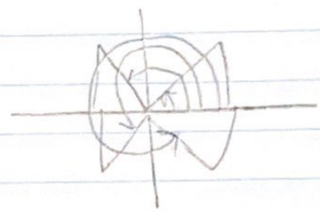
1a) $2\sin^2 x - 1 \leq 0$



$2\sin^2 x - 1 = 0$

$\sin^2 x = 1/2$

$\sin x = \pm \frac{\sqrt{1}}{\sqrt{2}} = \pm \frac{1}{\sqrt{2}}$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

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$0 \quad \frac{\pi}{4} \quad \frac{3\pi}{4} \quad \frac{5\pi}{4} \quad \frac{7\pi}{4} \quad 2\pi$

Intervals

Intervals	test points	$2\sin^2 x - 1 \leq 0$
$(0, \pi/4)$	$\pi/8$	T
$(\frac{\pi}{4}, 3\pi/4)$	$\pi/2$	F
$(\frac{3\pi}{4}, \frac{5\pi}{4})$	π	T
$(\frac{5\pi}{4}, \frac{7\pi}{4})$	$\frac{3\pi}{2}$	F
$(\frac{7\pi}{4}, 2\pi)$	$\frac{15\pi}{8}$	T

$\therefore, [0, \pi/4] \cup [3\pi/4, 5\pi/4] \cup [\frac{7\pi}{4}, 2\pi]$

Check endpoints

endpoints	$2\sin^2 x - 1 \leq 0$
0	T
$\frac{\pi}{4}$	T
$3\pi/4$	T
$5\pi/4$	T
$7\pi/4$	T
2π	T

1b) $\tan x < 2\sin x$

$\tan x = 2\sin x$

$\tan x - 2\sin x = 0$

$\frac{\sin x}{\cos x} - 2\sin x = 0$

$\sin x\left(\frac{1}{\cos x} - 2\right) = 0$

$\sin x = 0$

$x = 0, \pi$

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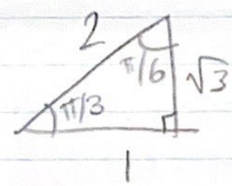
$\frac{1}{\cos x} - 2 = 0$

$\frac{1}{\cos x} = 2$

$1 = 2\cos x$

$1/2 = \cos x$

$x = \pi/3$



Intervals →

$0 \quad \pi/3 \quad \pi$

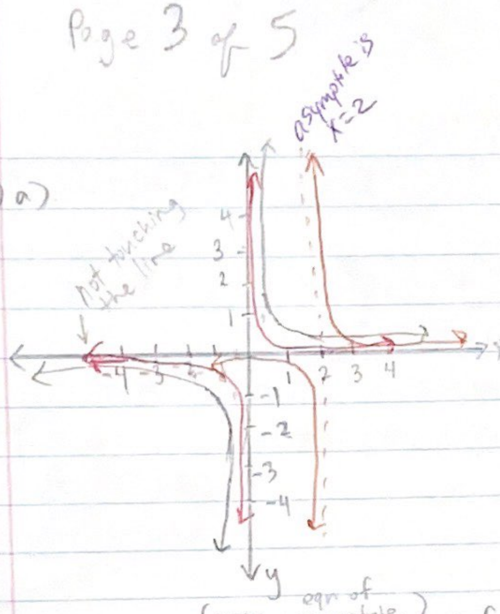
Intervals	test points	$\tan x < 2\sin x$
$(0, \pi/3)$	$\pi/6$	T
$(\pi/3, \pi)$	$2\pi/3$	T

test endpoints	Endpoints $\tan x < 2\sin x$
0	F
$\pi/3$	F
π	F

$\therefore (0, \pi/3) \cup (\pi/3, \pi)$

2) a)

$$y = \frac{1}{2x-4} = \frac{1}{2(x-2)}$$



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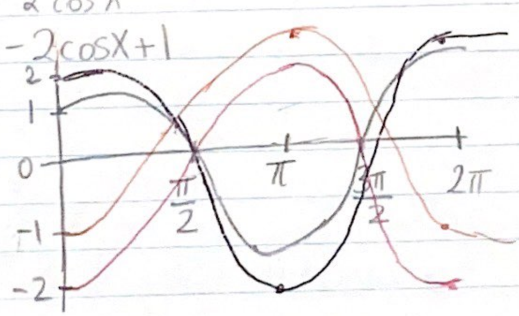
① • horizontal stretch by factor of 1/2
② • horizontal shift right 2 units
• orange color graph → final graph

2b) $y = 1 - 2\cos x$
$= -2\cos x + 1$



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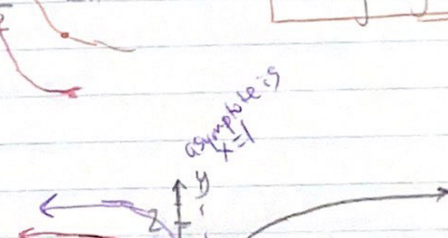
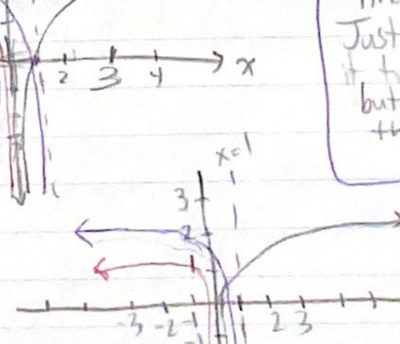
① Vertical stretch by factor of 2
② Vertical reflection (about x axis)
③ Vertical shift up by 1 unit
• orange graph → final graph

2c) $y = \ln(1-x)$
$y = \ln(-x+1)$
$y = \ln(-1(x-1))$



• purple graph = final. Just made it twice but same thing



2 +1

① Horizontal reflection (about y-axis)
② Horizontal shift one right

3) a) $f(x) = x^2+4x+2$

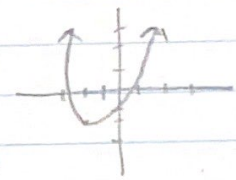
$f(-x) = (-x)^2+4(-x)+2$
$= x^2-4x+2$

$\therefore$, this function is not odd or even
(It is not symmetric about the y-axis)

Complete the square to find vertex

$y=(x^2+4x)+2$
$y=(x^2+4x+4)+2-4$
$y=(x^2+4x+4)-2$
$y=(x+2)^2-2$

vertex is $(-2,-2)$



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Periodic → No
Even → No
Odd → No
One to one → No (doesn't pass horizontal line test)
Interval of Increase → $[-2, \infty)$
Interval of decrease → $(-\infty, -2]$

3b) $\tan 2x$ for $x \in (-3\pi/4, 3\pi/4)$

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Periodic → yes
Odd → yes
even → No
One to one → No
Interval of Increase → $(-3\pi/4, -\pi/4) \cup (-\pi/4, \pi/4) \cup (\pi/4, 3\pi/4)$
Interval of decrease → None

$f(-x) = \tan 2(-x)$
$= \tan(-2x)$
$= -(\tan 2x)$
$f(-x) = -f(x) \rightarrow$ odd

4a) $f+g = \dfrac{x}{2x-1} + (\tan x + e^x)$

$fg = \left(\dfrac{x}{2x-1}\right)(\tan x + e^x)$

$f+g = \dfrac{x}{2x-1} + \tan x + e^x$

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$f \circ g = f(g(x)) = f(\tan x + e^x) = \dfrac{\tan x + e^x}{2(\tan x + e^x) - 1}$

4b) $f+g = (5x^2-2x)+(\sqrt[3]{1+x}-2)$ ✓

$fg = (5x^2-2x)(\sqrt[3]{1+x}-2)$ ✓

$f \circ g = f(g(x)) = f(\sqrt[3]{1+x}-2)$

$= 5(\sqrt[3]{1+x}-2)^2 - 2(\sqrt[3]{1+x}-2)$ ✓