$\frac{27}{30}$

Differential Calculus
03-62-139/140, Fall 2015
Lab Assignment 6

[redacted]

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1. Give equation(s) of the horizontal asymptote(s) and vertical asymptote(s) of the following curves.

   (a) (9 marks) $y = \dfrac{x^2 - 2x + 1}{x^2 - 6x + 5}$.

   (b) (8 marks) $y = \dfrac{2^x + 1}{2^x - 8}$.

2. (5 marks) Use the intermediate value theorem to show that the equation $\cos^{-1} x = e^x - 2$ has a real root.

3. (8 marks) Find the parameters a and b such that the function is continuous over the entire real line.

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x > 3 \\ \log_2(x+1) + a + 1 & \text{if } 0 \le x \le 3 \\ ae^x - b\cos x & \text{if } x < 0 \end{cases}$$

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1) Vertical asymptotes

a) $y = \frac{x^2 - 2x + 1}{x^2 - 6x + 5}$

Set bottom equal to zero

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$x^2 - 6x + 5 = 0$

Factor $\rightarrow (x-1)(x-5)$

$x - 1 = 0$ $\quad$ $x - 5 = 0$

$x = 1$ $\quad$ $x = 5$ ✓

need to prove the VA and if they $= \infty$ its good ✓

The equations of the vertical asymptotes are $x = 1$ and $x = 5$

Horizontal Asymptotes

Take the limit at ∞ and $-\infty$

$\lim_{x\to\infty} \frac{x^2 - 2x + 1}{x^2 - 6x + 5}$

$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{6x}{x^2} + \frac{5}{x^2}} = \lim_{x\to\infty} \frac{1 - \frac{2}{x} + \frac{1}{x^2}}{1 - \frac{6}{x} + \frac{5}{x^2}} = 1$ ✓

$\lim_{x\to-\infty} \frac{x^2 - 2x + 1}{x^2 - 6x + 5}$

$= \lim_{x\to-\infty} \frac{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{6x}{x^2} + \frac{5}{x^2}} \rightarrow \lim_{x\to-\infty} \frac{1 - \frac{2}{x} + \frac{1}{x^2}}{1 - \frac{6}{x} + \frac{5}{x^2}}$

$= \pm 1$

$\therefore y = 1$ and $y = -1$ are horizontal asymptotes

1b) Vertical Asymptotes

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$y = \frac{2^x + 1}{2^x - 8}$

Set denominator $= 0$

$2^x - 8 = 0$

$2^x = 8$

$x = 3 \rightarrow$ equation of vertical asymptote

1b) Horizontal Asymptotes:

$$\lim_{x\to\infty} \frac{2^x+1}{2^x-8} = \lim_{x\to\infty} \frac{\frac{2^x}{2^x}+\frac{1}{2^x}}{\frac{2^x}{2^x}-\frac{8}{2^x}} = \lim_{x\to\infty} \frac{1+1/2^x}{1-8/2^x} = \frac{1+0}{1-0} = 1$$

$$\lim_{x\to-\infty} \frac{2^x+1}{2^x-8} = \lim_{x\to-\infty} \frac{\cancel{2^x}+1}{\cancel{2^x}-8} = \frac{1}{-8}$$

∴, the eqns of the horizontal asymptotes are $y=1$ and $y=\frac{-1}{8}$

5

2) $\cos^{-1}x = e^x - 2$

$\cos^{-1}x - e^x + 2 = 0$

Let $f(x) = \cos^{-1}x - e^x + 2$

Let $[a,b] = [0,1]$

$f(0) = \cos^{-1}(0) - e^0 + 2 = 2.57 \quad > 0$

$f(1) = \cos^{-1}(1) - e^{(1)} + 2 = -0.718 \quad < 0$

There is a root in the interval $[0,1]$

For $x > 3$

$f(x) = \frac{x^2 - 9}{x-3}$ is not continuous at $x=3$

$\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$ hole @ $x=3$

For $x<3$, $ae^x - b\cos x$ is continuous over its domain (trig and exponential function)

For $x=3$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \log_2(x+1)+a+1 = \log_2(3+1) + a+1$

$= \log_2(4) + a+1$

$= 2 + a+1$

$= 3+a$

$\lim_{x\to 3^+} f(x) \; \frac{x^2-9}{x-3} = \lim_{x\to 3^+} f(x) \; \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$ hole at $x=3$

$= \lim_{x\to 3^+} f(x) \; (x+3) = 3+3 = 6$

Continuous if $\lim_{x\to 1^+} = \lim_{x\to 1^-} = f(3)$

$6 = 3+a = 3+a$

$6 = 3+a$

$\boxed{a=3}$

$f(3) = \log_2(3+1) + a+1$

$= \log_2(4) + a+1$

$= 3+a$

For b $\rightarrow a=3$

For $x \geq 0$

$\log_2(x+1) + \underset{3}{a} + 1$

$= \log_2(x+1)+4$ is continuous over its domain because it is a log function

For $x<0$, $ae^x - b\cos x$ is continuous over its domain because it is a trigtexponential function

For $x=0$

$\lim_{x\to 0^-} f(x)$ $ae^x - b\cos x = \lim_{x\to 0^-} f(x)$ $3e^x - b\cos x = 3e^0 - b\cos 0$

$= 3(1) - \cos 0$

$4 = 3 - b$

$\boxed{b=-1}$

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$\lim_{x\to 0^+} f(x)$ $\log_2(x+1) + 4$ $= \log_2(0+1)+4$ $= \log_2(1)+4 = 0+4 = 4$

$a+1 = 3+1$

$\therefore$, $a=3$ and $b=-1$