[redacted]

DEPARTMENT OF MATHEMATICS AND STATISTICS
Differential Calculus 62-139/140
Midterm Exam 1
Saturday, October 24, 2015

[redacted]

9~~8~~%

96%

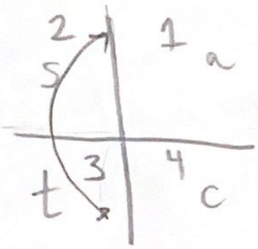
Instructions :

- This test has 8 problems and a total of 7 pages, including this cover page. You have 80 minutes.
- Read carefully and answer **all** questions. Show all the work to receive full credit.
- Only non-graphing and non-programmable calculators are permitted.
- Work all problems in the space provided.
- You must give **exact** answers (and not decimal approximations).

(5) 1. If $\sin x = \frac{2}{3}$ and $\cot y = \frac{5}{3}$, where x and y lie between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, evaluate $\cos(x+y)$

$\cos(A+B)$ or $\cos(x+y)$

$= \cos A \cos B - \sin A \sin B$ or $\cos x \cos y - \sin x \sin y$



$\sin x = \frac{2}{3}$

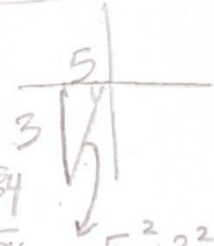
$\cos x = -\frac{\sqrt{5}}{3}$

$\cot y = 5/3$

$\tan y = \frac{3}{5}$

$\cos y = -5/\sqrt{34}$

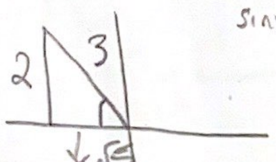
$\sin y = -3/\sqrt{34}$



$5^2 + 3^2 = c^2$

$c = \sqrt{34}$

$\sin x = 2/3$



$3^2 - 2^2 = c^2$

$c = \sqrt{5}$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$= \left(-\sqrt{5}/3\right)\left(-5/\sqrt{34}\right) - \left(2/3\right)\left(-3/\sqrt{34}\right)$

$= \boxed{= \frac{5\sqrt{5}+6}{3\sqrt{34}}}$

(3) 2. Find all values of x in the interval $[0, \pi]$ that satisfy the inequality

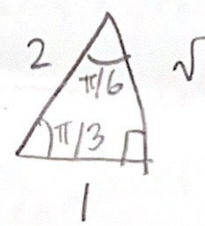
$$2\cos x - 1 > 0$$

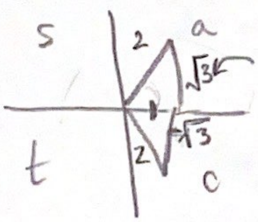
$[0, \pi]$

where's π

$2\cos x - 1 = 0$

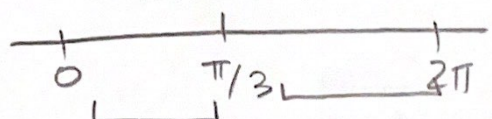
$\cos x = 1/2 \rightarrow x = \pi/3$





$2^2 - 1^2 = c^2$

$c = \sqrt{3}$



$(0, \pi/3)$ $(\pi/3, 2\pi)$

endpoints	$2\cos x - 1 > 0$
0	T
$\pi/3$	F

Intervals	test points	$2\cos x - 1 > 0$
$(0, \pi/3)$	$\pi/6$	T
$(\pi/3, 2\pi)$	π	F

$\therefore$, the values are $[0, \pi/3)$

(5) 3. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation

$$\cos 2x = 3 - 5\sin x$$

$\cos 2A = 1 - 2\sin^2 A$

$\cos 2x = 3 - 5\sin x$

$1 - 2\sin^2 x = 3 - 5\sin x$

$1 - 2\sin^2 x - 3 + 5\sin x = 0$

$-2\sin^2 x - 2 + 5\sin x = 0$

$-2\sin^2 x + 5\sin x - 2 = 0$

let $\sin x = x_1$

$-2x_1^2 + 5x_1 - 2 = 0$

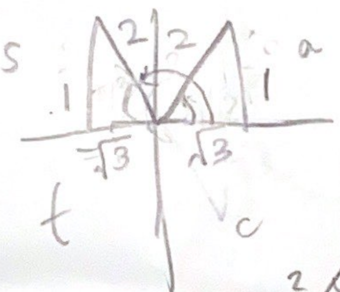
$\dfrac{-5 \pm \sqrt{25 - 4(-2)(-2)}}{(-2)(2)}$

$= \dfrac{-5 \pm 3}{-4}$ $-2/-4 = 1/2$; 2

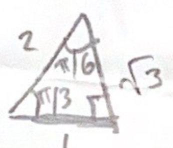
Why not Sin?

$\cancel{\cos x} = 2 \rightarrow$ Not possible

$\sin x = 1/2$



$\boxed{x = \pi/6, \frac{5\pi}{6}}$



(6) 4. Find a formula for the inverse of the given function. Also find the domain and range of f and f^{-1}.

6

$$f(x) = \frac{3x + 6}{-7x + 2}$$

$y = \dfrac{3x+6}{-7x+2}$

$x = \dfrac{3y+6}{-7y+2}$

$(-7y+2)(x) = 3y+6$

$-7yx + 2x - 3y = 6$

$-7yx - 3y = 6 - 2x$

$y(-7x - 3) = 6 - 2x$

$\boxed{f^{-1}(x) = \dfrac{6-2x}{(-7x-3)}}$

Domain of $f(x) = \{x \in \mathbb{R} \mid x \neq 2/7\}$

$-7x + 2 = 0$
$x = -2/-7$ $x = 2/7$

Range of $f(x) = \{y \in \mathbb{R} \mid y \neq 3/-7\}$

Domain of $f^{-1}(x) = \{x \in \mathbb{R} \mid x \neq 3/-7\}$

Range $f^{-1}(x) = \{y \in \mathbb{R} \mid y \neq 2/7\}$

4

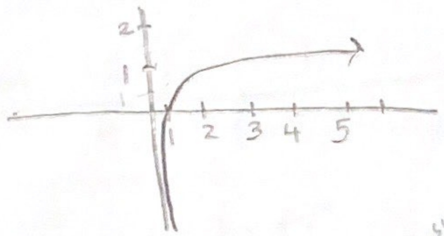
(4) 5. Graph the function $y = 1-\ln(x+3)$ by hand, not by plotting points, but by starting with the graph of one of the standard functions and applying the appropriate transformations.

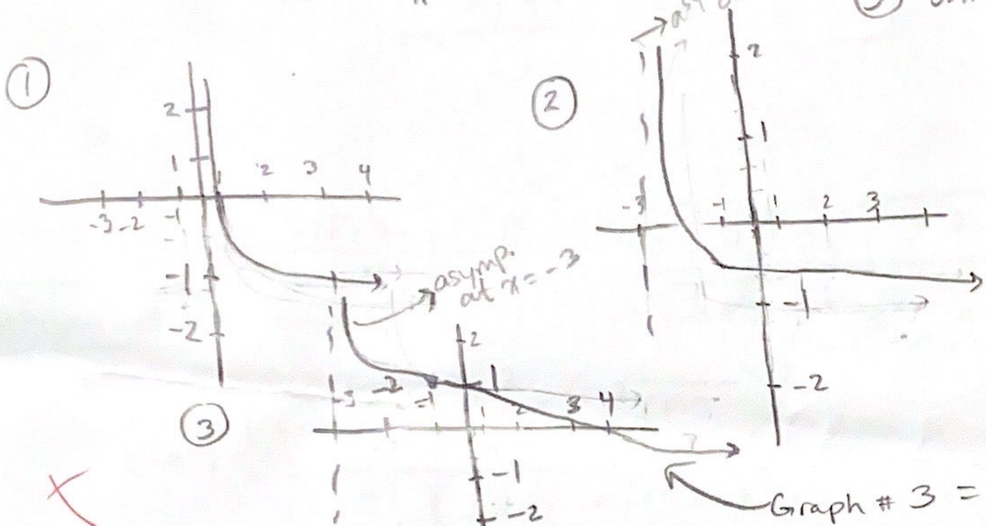
$y = -\ln(x+3)+1$

① Vertical reflection

② shift left 3

③ shift up 1





Graph # 3 = final answer

(5) 6. Solve $\log_3 x + \log_3(x-8) = 2$

$\log_3(x)(x-8)=2$

practice ↳

$3^{x(x-8)} = 3^2$

$x^2 - 8x = 9$

$x^2 - 8x - 9 = 0$

$$\frac{8 \pm \sqrt{64 - 4(1)(-9)}}{2}$$

$$= \frac{8 \pm 10}{2} \quad 9, \; -1$$

$x \neq -1$

So $\boxed{x=9}$

(15) 7. Evaluate the following limits, if they exist.

(a) $\lim\limits_{x\to 0} \dfrac{\cos(x) - \ln(1+2x)}{4e^{5x}\tan(x) + x^2 + 1}$

$$= \lim_{x\to 0} = \frac{\cos(0) - \ln(1+2(0))}{4e^{5(0)}\tan(0) + (0)^2 + 1}$$

$$= \frac{1 - \cancel{\ln(1)}^{0}}{1} = \boxed{1}$$

2

(b) $\lim\limits_{x\to 7} \dfrac{\sqrt{2x+5} - \sqrt{x+12}}{x^2 - 49} \cdot \dfrac{\sqrt{2x+5} + \sqrt{x+12}}{\sqrt{2x+5} + \sqrt{x+12}}$

Sub, you get 0 so rationalize

$$= \lim_{x\to 7} = \frac{(\sqrt{2x+5})^2 - (\sqrt{x+12})^2}{(x^2-49)[(\sqrt{2x+5} + \sqrt{x+12})]}$$

$$= \lim_{x\to 7} \frac{2x+5-(x+12)}{(x+7)(x-7)[(\sqrt{2x+5}) + (\sqrt{x+12})]}$$

$$= \lim_{x\to 7} \frac{\cancel{(x-7)}}{(x+7)\cancel{(x-7)}[(\sqrt{2x+5}) + (\sqrt{x+12})]}$$ hole at $x=7$

$$= \lim_{x\to 7} \frac{1}{(x+7)[(\sqrt{2x+5}) + \sqrt{x+12}]}$$ hole at $x=7$

5

$$= \frac{1}{(7+7)(\sqrt{2(7)+5} + \sqrt{7+12})}$$

$$= \frac{1}{(14)(\sqrt{19} + \sqrt{19})} = \frac{1}{(14)(2\sqrt{19})} = \boxed{\frac{1}{28\sqrt{19}}}$$

(c) $\lim_{x\to 0} x^6 \sin\left(\frac{20}{x^3}\right)$

4

$= \cancel{\lim_{x\to 0}} \left(-1 \le \sin\left(\frac{20}{x^3}\right) \le 1\right)$ ✓

$= \cancel{\lim_{x\to 0}} \left(-x^6 \le x^6 \sin\frac{20}{x^3} \le x^6\right)$ ✓ ← apply x^6 to both sides

$= \lim_{x\to 0} -x^6 \le \lim_{x\to 0} x^6 \sin\frac{20}{x^3} \le \lim_{x\to 0} x^6$ ✓

$= 0 \le \lim_{x\to 0} x^6 \sin\frac{20}{x^3} \le 0$ ✓

$\boxed{\therefore\text{, limit } = 0}$ ✓

* Practice (d) $\lim_{x\to -\infty} \frac{3x+7}{\sqrt{16x^2+8}}$

4

$$\lim_{x\to -\infty} \frac{\frac{3x}{x}+\frac{7}{x}}{\sqrt{\frac{16x^2}{x^2}+\frac{8}{x^2}}}$$

$$= \lim_{x\to -\infty} \frac{3+\frac{7}{x}}{\sqrt{16+8/x^2}}$$

$$= \lim_{x\to -\infty} -\left[\frac{3+0}{\sqrt{16+0}}\right]$$

$$\lim_{x\to -\infty} -\left[\frac{3}{\sqrt{16}}\right]$$

$\therefore$, limit is

$-\frac{3}{4}$

(7) 8. For each of the following, circle either TRUE or FALSE.

(a) (T/F) The graph of $x = y^2 + 2$ is a parabola.

(b) (T/F) $f(x) = \left(\frac{1}{2}\right)^x$ is an increasing function.

(c) (T/F) X If $f(x) = x^2$ and $g(x) = \frac{1}{x+1}$ then $(f \circ g)(x) = \frac{1}{x^2+1}$.

(d) (T/F)The function $f(x) = \tan x$ is a periodic function on $\mathbb{R}$

(e) (T/F) $f(x) = x^3 + 1$ is an odd function.

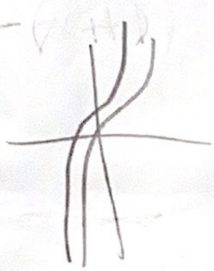
(f) (T/F) The function $f(x) = e^x$ is a one-to-one function.

(g) (T/F) X The domain of the function $f(x) = \sin^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. $[-1,1] \leftarrow$ range of $\sin x$ = domain of $\sin^{-1} x$

$-f(x)$

$f(x)$

$f(-x) = (-x)^3 + 1$



$f\left(\frac{1}{x+1}\right)$

$\left(\frac{1}{x+1}\right)^2$

$= \frac{1^2}{x^2+1}$

Question	Points	Score
1	5	5
2	3	3
3	5	5
4	6	6
5	4	4
6	5	5
7	15	15
8	7	5
Total:	50	48