MATH 2030, F16 EXAM #1 [redacted]

Name: [redacted]

Instructions:

1. Read each question carefully and BOX your answers.
2. You must show ALL work to receive credit. Correct answers without sufficient work may receive no credit.
3. You may use your calculator (if it helps!).

Problem	Points	Max
1	10	10
2	10	10
3	10	10
4	10	10
5	10	10
Total	50	50

100%

10

1. Find an equation of the line that contains the point $(1, 0, 6)$ and which is perpendicular to the plane $x + 3y + z = 17$.

$\langle 1,0,6 \rangle$

$\langle \text{point} \rangle + t \langle \text{vector } \vec{n} \rangle$

$= \langle 1, 3, 1 \rangle$

coefficients are the $\vec{n}$

$(1,0,6) = 1\vec{i} + 0\vec{j} + 6\vec{k} = \vec{i} + 6\vec{k}$

$(\vec{i} + 6\vec{k}) + t\langle 1,3,1 \rangle = 0 \rightarrow$ vector eqn

$(\vec{i} + 6\vec{k}) + t(\vec{i} + 3\vec{j} + \vec{k}) = 0$

$(\vec{i} + 6\vec{k}) + t\vec{i} + 3t\vec{j} + t\vec{k} = 0$

$t\vec{i} + \vec{i} + 6\vec{k} + t\vec{k} + 3t\vec{j} = 0$

$(t+1)\vec{i} + (3t)\vec{j} + (6+t)\vec{k} = 0$

$$\left.\begin{aligned} x &= t+1 \\ y &= 3t \\ z &= 6+t \end{aligned}\right\} \text{parametric eqn}$$

$$\left.\begin{aligned} t &= x-1 \\ t &= \frac{y}{3} \\ t &= z-6 \end{aligned}\right\} \rightarrow \left\{\frac{x-1}{1} = \frac{y}{3} = \frac{z-6}{1}\right\} \text{symmetric eqn}$$

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2. Determine whether the lines are parallel, skew or intersecting. If intersecting, determine the point of intersection.

$$L_1 : x = 1 + 2t, \quad y = -1 - 4t, \quad z = 3t$$
$$L_2 : x = 2 - s, \quad y = 3s, \quad z = 1 + s$$

Intersecting?

$X = 1 + 2t$

$X = 2 - S$

$1 + 2t = 2 - S$

$t = \frac{2 - S - 1}{2} = \frac{1 - S}{2}$

$y = -1 - 4t$

$y = 3S$

$-1 - 4t = 3S$

$t = \frac{3S + 1}{-4}$

Parallel?

$\langle 2, -4, 3 \rangle$

$\langle -1, 3, 1 \rangle$

Not scalar multiples, there is no number you can multiply either of the vectors by to get the other vector

Not parallel

$\frac{1-S}{2} = \frac{3S+1}{-4}$

$-4(1-S) = 2(3S+1)$

$-4 + 4S = 6S + 2$

$-4 - 2 = 6S - 4S$

$-6 = 2S$

$\frac{-6}{2} = S$

$S = -3$

$t = \frac{1-S}{2} = \frac{1-(-3)}{2}$

$= \frac{1+3}{2} = \frac{4}{2} = 2$

$t = 2$
$S = -3$ plug in

Check by subbing in S and t

y

$-1 - 4t = 3S$

$-1 - 4(2) = 3(-3)$

$-1 - 8 = -9$

$-9 = -9$ ✓

They equal here but in the z eqn they do not, both eqns must equal in all 3 eqns to intersect or there is no point of intersection

z

$z = 3t$

$z = 1 + S$

$3t = 1 + S$

$3(2) = 1 + (-3)$

$6 = 1 - 3$

$6 \neq -2$

x

$1 + 2t = 2 - S$

$1 + 2(2) = 2 - (-3)$

$1 + 4 = 5$

$5 = 5$

They equal here, but for the z equation they do not. They must equal in all equations to intersect

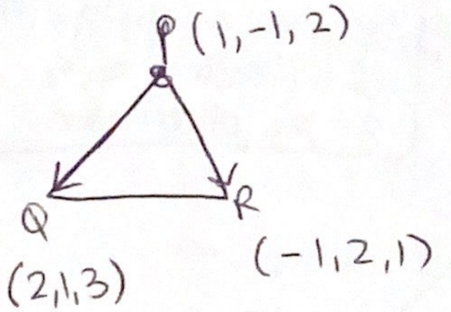
$\therefore$, they are not intersecting and they are skew lines

10 **3.** Find an equation of the plane passing through the points $P = (1, -1, 2)$, $Q = (2, 1, 3)$, $R = (-1, 2, 1)$.

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$(x_0, y_0, z_0) \Rightarrow$ point

$\langle a, b, c\rangle = \vec{n}$



$\overrightarrow{PR} = \langle -1-1,\ 2-(-1),\ 1-2\rangle = \langle -2, 3, -1\rangle$

$\overrightarrow{PQ} = \langle 2-1,\ 1-(-1),\ 3-2\rangle = \langle 1, 2, 1\rangle$

$\overrightarrow{PR} \times \overrightarrow{PQ}$ to get $\vec{n}$ (normal vector orthogonal to plane)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$\overrightarrow{PR} = \langle -2, 3, -1\rangle$

$\overrightarrow{PQ} = \langle 1, 2, 1\rangle$

$$\Rightarrow [(3)(1)-(-1)(2)]\vec{i} - [(-2)(1)-(-1)(1)]\vec{j} + [(-2)(2)-(3)(1)]\vec{k}$$

$$= (3-(-2))\vec{i} - (-2+(+1))\vec{j} + (-4-3)\vec{k}$$

$$= 5\vec{i} + (+1)\vec{j} + (-7)\vec{k}$$

$$\vec{n} = \langle 5, 1, -7\rangle$$

Pick any point, say $P = (1, -1, 2)$

$$\boxed{\text{Eqn: } 5(x-1) + 1(y+1) - 7(z-2) = 0}$$

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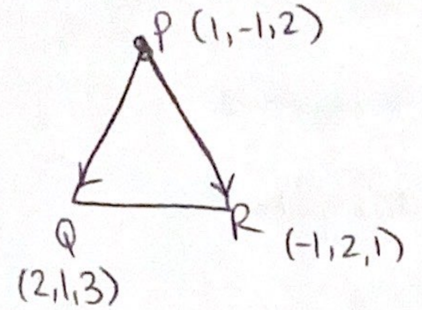
4. Find the area of triangle PQR, where the points P, Q, R are given in problem 3.

Area of triangle $\rightarrow \dfrac{\text{magnitude of cross product}}{2}$



$\overrightarrow{PQ} = \langle 2-1,\ 1-(-1),\ 3-2 \rangle = \langle 1, 2, 1 \rangle$

$\overrightarrow{PR} = \langle -1-1,\ 2-(-1),\ 1-2 \rangle = \langle -2, 3, -1 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ -2 & 3 & -1 \end{vmatrix} = [(2)(-1) - (3)(1)]\vec{i} - [(1)(-1) + (+2)(1)]\vec{j} + [(1)(3) - (-2)(2)]\vec{k}$$

$$= (-2-3)\vec{i} - (-1+2)\vec{j} + (3+4)\vec{k}$$

$$= -5\vec{i} - (1)\vec{j} + 7\vec{k}$$

$$\cancel{\$}\ \overrightarrow{PQ} \times \overrightarrow{PR} = \langle -5, -1, 7 \rangle$$

$$\| \overrightarrow{PQ} \times \overrightarrow{PR} \| = \sqrt{(-5)^2 + (-1)^2 + (7)^2} = \sqrt{25+1+49} = \sqrt{75}$$

Area of triangle PQR $= \dfrac{\sqrt{75}}{2}$

Extracredit

Range of the triangle $\rightarrow$ the y values

constitute range, since the triangle is in 2D you can look at the y-values

Range $= \{y \in \mathbb{R} \mid -1 \le y \le 2\}$

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5. Find the angle between the planes

$$3x + 6z = 1 \quad \text{and} \quad 2x + 2y - z = 3.$$

$$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{\|\vec{n_1}\| \, \|\vec{n_2}\|}$$

$\vec{n_1} = \langle 3, 0, 6 \rangle$ → coefficients from equation 1

$\vec{n_2} = \langle 2, 2, -1 \rangle$ → coefficients from equation 2

$\vec{n_1} \cdot \vec{n_2} = \langle 3,0,6 \rangle \cdot \langle 2,2,-1 \rangle = (3)(2) + (0)(2) + (6)(-1)$

$= 6 + 0 - 6 = 6 - 6 = 0$

$\cos\theta = \dfrac{0}{\|\vec{n_1}\| \, \|\vec{n_2}\|}$ means the whole right side is 0

$\cos\theta = 0$

$\boxed{\theta = 90^\circ}$ (the angle between the planes is 90°, they're perpendicular because dot product is 0 (between the normal vectors))