MATH 2030 F16 [redacted] EXAM #2 [redacted]

Name: [redacted]

Instructions:

1. Read each question carefully and BOX your answers.
2. You must show ALL work to receive credit. Correct answers without sufficient work may receive no credit.
3. You may use your calculator (if it helps!).

Problem	Points	Max
1	5	10
2	10	10
3	10	10
4	8	10
5	10	10
Total	43	50

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$R = \dfrac{V_0^2 \sin 2\theta}{g}$

$\Delta x = V_0 \cos\theta\, t$

$\Delta y = V_0 \sin\theta\, t - \frac{1}{2} g t^2$

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1. A particle starts at the origin with initial velocity $\langle 1, -1, 3\rangle$. Its acceleration is $\mathbf{a}(t) = \langle 6t, 12t^2, -6t\rangle$. Find its position function $\mathbf{r}(t)$.

$$\int a(t) = v(t)$$

$$\int v(t) = r(t)$$

$$\int \langle 6t, 12t^2, -6t\rangle = \langle \frac{\cancel{6}^{3}t^2}{\cancel{2}}, \frac{\cancel{12}^{4}t^3}{\cancel{3}}, -\frac{\cancel{6}^{3}t^2}{\cancel{2}}\rangle + C$$

$$v(t) = \langle 3t^2, 4t^3, -3t^2\rangle = \langle 1, -1, 3\rangle$$

Constants?

$$\int \langle 3t^2, 4t^3, -3t^2\rangle \Rightarrow \langle \frac{\cancel{3}t^3}{\cancel{3}}, \frac{\cancel{4}t^4}{\cancel{4}}, -\frac{\cancel{3}t^3}{\cancel{3}}\rangle$$

$$r(t) = \langle t^3, t^4, -t^3\rangle$$

Rough work

Find parameter t?

$1 = 3t^2 \rightarrow t = \pm\sqrt{\frac{1}{3}}$

$-1 = 4t^3 \rightarrow t = \sqrt[3]{-\frac{1}{4}} \Rightarrow -\frac{1}{\sqrt[3]{4}}$

$3 = -3t^2 \rightarrow t = \pm\sqrt{-1}$

$r(t) = \langle x_0, y_0, z_0\rangle + t\langle x, y, z\rangle$

10 **2.** Find the linear approximation of the function $f(x, y) = \sqrt{x^2 + y^2}$ at the point $(3, 4)$ and use it to estimate the number $\sqrt{(3.01)^2 + (3.97)^2}$.

$$L(x,y) = f(a,b) + f_x(a,b)(x - x_0) + f_y(a,b)(y - y_0)$$

$$f(a,b) = f(3,4) = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = 5$$

$$f_x \text{ of } (x^2+y^2)^{1/2} \Rightarrow \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x$$

$$f_y \text{ of } (x^2+y^2)^{1/2} \Rightarrow \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2y$$

$$f_x \text{ at } (\underset{x}{3}, \underset{y}{4}) \Rightarrow \frac{1}{2}(3^2+4^2)^{-1/2} \cdot 2(3)$$

$$= \frac{1}{2} \cdot (25)^{-1/2} \cdot 6$$

$$= \frac{6}{2} \cdot (25)^{-1/2} = 3\left(\frac{1}{\sqrt{25}}\right) = \frac{3}{5} = f_x$$

$$f_y \text{ at } (3,4) = \frac{1}{2}(3^2+4^2)^{-1/2} \cdot (2)(4)$$

$$= \frac{1}{2}(25)^{-1/2} \cdot 8 = \frac{8}{2} \cdot (25)^{-1/2} = 4 \cdot \frac{1}{5} = \frac{4}{5} = f_y$$

$$L(x,y) = 5 + \frac{3}{5}(x-3) + \left(\frac{4}{5}\right)(y-4) \quad \text{at } (3.01, 3.97)$$

$$L(3,4) = 5$$

$$L(3.01, 3.97) = 5 + \frac{3}{5}(3.01 - 3) + \frac{4}{5}(3.97 - 4)$$

$$= 5 + \frac{3}{5}(0.01) + \frac{4}{5}(-0.03) = 4.982$$

10 **3.** Find the equation of the *tangent plane* to the surface

$$z = e^x \cos y$$

at the point $(0, 0, 1)$.
$\quad\;\; x \;\; y \;\; z$

$$z - z_0 = (x - x_0)(f_x(a,b)) + (y - y_0)(f_y(a,b))$$

$$z - 1 = (x - 0)(f_x(a,b)) + (y - 0)(f_y(a,b))$$

f_x of $e^x \cos y \Rightarrow (e^x)'(\cos y) + \cancel{(e^x)(\cos y)'}^{\,0}$

$f_x \Rightarrow (e^x)(\cos y)$ at $(0,0,1)$ is
$\Rightarrow (e^0)(\cos 0) = (1)(1) = 1$

f_y of $e^x \cos y \Rightarrow f_y (e^x)(-\sin y)$

f_y at $(0,0,1) \rightarrow e^0 \cdot (-\sin(0)) = 1 \cdot 0 = 0$

$$z - 1 = (x)(1) + \cancel{(y)(0)}$$

$$z - 1 = x$$

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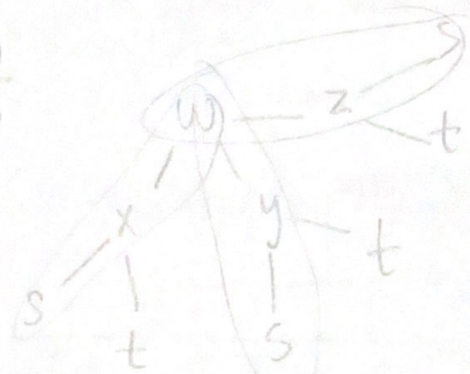
4. Suppose $w = xe^{y-z^2}$, where $x = 2st$, $y = s-t$ and $z = s+t$. Find

$$\frac{\partial w}{\partial s}$$

when $s = 2$, $t = -1$.

$\frac{dw}{ds}$



$$\frac{dw}{ds} = \frac{dw}{dx}\frac{dx}{ds} + \frac{dw}{dy}\frac{dy}{ds} + \frac{dw}{dz}\frac{dz}{ds}$$

$w = xe^{(y-z^2)}$

$x = 2st$

$y = s-t$

$z = s+t$

$\frac{dx}{ds} \Rightarrow 2(t)$ $\quad$ $\frac{dy}{ds} \Rightarrow 1$ $\quad$ $\frac{dz}{ds} = 1$

$\frac{dw}{dx} = \Rightarrow (x)'(e^{y-z^2}) \cancel{+ (x)(e^{y-z^2})'} \Rightarrow e^{y-z^2}$

$\frac{dw}{dy} \Rightarrow (x)(e^{y-z^2})' = (x)(e^{y-z^2})(y-z^2)' \Rightarrow (x)(e^{y-z^2})(1)$

$\frac{dw}{dz} = (x)(e^{y-z^2})' = (x)(e^{y-z^2})(y-z^2)' \Rightarrow (x)(e^{y-z^2})(-2z)$

$(e^{y-z^2})(2t) + (x)(e^{y-z^2}) + (x)(e^{y-z^2})(-2z$

$\Rightarrow s = 2$
$t = -1$

$(e^{y-z^2})(2(-1)) + (x)(e^{y-z^2}) + (x)(e^{y-z^2})(-2z)$

$\Rightarrow e^{y-z^2}\left[-2 + x + (-2z)(x)\right]$

$x, y, z = ?$

10 **5.** Find the length of the curve

$$\mathbf{r}(t) = \langle 2t^{3/2}, \cos 2t, \sin 2t \rangle, \quad \overset{a}{0} \le t \le \overset{b}{1}.$$

$\int_a^b \|\vec{r}\,'(t)\|$ = arc length

$\int_0^1$

$\vec{r}\,'(t) = \langle 3t^{1/2}, -2\sin 2t, 2\cos 2t \rangle$

$\|\vec{r}\,'(t)\| = \sqrt{(3\sqrt{t})^2 + (-2\sin 2t)^2 + (2\cos 2t)^2}$

$= \sqrt{9t + (4(\sin 2t)^2) + (4(\cos 2t)^2)}$

$= \sqrt{9t + 4[(\sin 2t)^2 + (\cos 2t)^2]}$

$= \sqrt{9t + 4(1)}$

$= \sqrt{9t+4}$

$\int_0^1 \sqrt{9t+4}\, dt \longrightarrow$ let $u = 9t+4$

$du = 9\,dt$

$\frac{du}{9} = dt$

$\frac{1}{9}\int_0^1 \sqrt{u}\, du$

$\frac{1}{9}\left(\frac{u^{3/2}}{3/2}\right) \Rightarrow \frac{2}{3 \cdot 9}\left((9t+4)^{3/2}\right)\Big]_0^1 \Rightarrow \frac{2}{27}\left[(9(1)+4)^{3/2} - (9(0)+4)^{3/2}\right]$

$\frac{2}{27}\left[(9+4)^{3/2} - (4)^{3/2}\right]$

$\frac{2}{27}\left[(\sqrt{13})^3 - 8\right]$