MATH 2030 F16 [redacted] EXAM #3 [redacted]

[redacted]

Instructions:

1. Read each question carefully and BOX your answers.
2. You must show ALL work to receive credit. Correct answers without sufficient work may receive no credit.
3. You may use your calculator (if it helps!).

Problem	Points	Max
1	10	10
2	16	10
3	5	10
4	5	10
5	10	10
Total	40	50

10

1. Find the directional derivative of the function $f(x, y) = x^2 e^y$ at the point $(-2, 0)$ in the direction given by the unit vector which makes a 60 degree angle with the x-axis.

$f_x =$ e^5

$\cancel{(x^2)(e^y)' + (x^2)'(e^y)}$

$\cancel{2(x^2)(e^y)}$ $(2x)(e^y)$

$f_y = (x^2)(e^y)$

$f_x = (2x)(e^y)$ at $(-2,0)$ $=$ $(2)(-2)(e^0) \Rightarrow -4$

$f_y = x^2 e^y$ at $(-2,0)$ $=$ $(-2)^2(e^0) \Rightarrow (4)(1) = 4$

$(-4, 4) \cdot (\cos 60, \sin 60)$

$\langle -4, 4\rangle \cdot \langle 0.5, \frac{\sqrt{3}}{2} \rangle$

$D_u f = \vec{\nabla} f \cdot \vec{u}$

$\Rightarrow (-4)(\frac{1}{2})\hat{\imath} + (4)(\frac{\sqrt{3}}{2})\hat{\jmath}$

$\Rightarrow \frac{-4}{2}\hat{\imath} + \frac{4\sqrt{3}}{2}\hat{\jmath}$

$= -2\hat{\imath} + 2\sqrt{3}\hat{\jmath}$

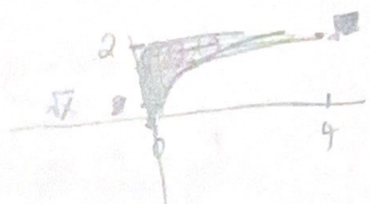
$= \cancel{\langle -2, 2\sqrt{3} \rangle}$

$-2 + 2\sqrt{3}$

10

2. Evaluate the integral by reversing the order of integration

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx\,.$$



$$\int_0^2 \left(\int_0^{y^2} \frac{1}{y^3+1}\, dx \right) dy$$

y is like a constant, integrate wrt x first

$$\int_0^2 \left. \frac{x}{y^3+1} \right]_0^{y^2} dy$$

$$= \int_0^2 \left(\frac{y^2}{y^3+1} - \cancel{\frac{0}{y^3+1}} \right) dy$$

$$\int_0^2 \frac{y^2}{y^3+1}\, dy$$

$u = y^3+1$

$du = 3y^2\, dy$

$\frac{du}{3} = y^2\, dy$

$$\int_0^2 \frac{1}{u} \cdot \frac{du}{3}$$

$$= \frac{1}{3} \int_0^2 \frac{1}{u}\, du$$

$$= \frac{1}{3} \left[\ln(y^3+1) \right]_0^2$$

$$= \frac{1}{3} \left(\ln(9) - \underbrace{\ln(1)}_{0} \right)$$

$\frac{1}{3} \ln 9$

A: Find fx, fy, = 0, solve (-2,3) point

5

3. Find the local maxima, local minima and saddle points of the function

$$f(x, y) = x^2 + xy + y^2 + x - 4y + 5.$$

$f_x = 2x + y + 1$ ✓

$f_y = x + 2y - 4$ ✓

$f_{xy} = 1$

$f_{yx} = 1$

1 + 1 + 1 - 1 - 4 + 5
6

4 + 4 + 4 + 4 - 8 + 5
16 - 8 = 8 + 5

$f_x = 0 \to 2x + y + 1 = 0$

$f_y = 0 \to x + 2y - 4 = 0$

4 cases

1) x, y both $= 0$

Point (0,0)

$0 = f_x = 0 + 0 + 1$

$0 = f_y = 0 + 0 - 4$

2) x, y both $\neq 0$

$2x + y + 1 = 0$

$x + 2y - 4 = 0$

Cannot get more information (x, y)

3) $x = 0,\ y \neq 0$

$f_x = 2(0) + y + 1 \to y + 1 = 0$ $y = -1$

$f_y = 0 + 2y - 4 \to 2y - 4 = 0$ $y = \frac{4}{2} = 2$

Points → ~~(0, -1)~~ ~~(0, 2)~~

Minimums, $D > 0$, $f_{xx} > 0$

4) $x \neq 0,\ y = 0$

$f_x = 0 = 2x + \cancel{0} + 1 \Rightarrow 2x + 1 = 0 \to x = -\frac{1}{2}$

$f_y = 0 = x + 2(0) - 4 \to x - 4 = 0$
$x = 4$

All points are minimums

No maximums

Points → $(-\frac{1}{2}, 0)$, $(4, 0)$

Minimum points, $D > 0$ ✓ $f_{xx} > 0$

$f_{xx} = 2$

$f_{yx} = f_{xy} = 1$

$f_{yy} = 2$

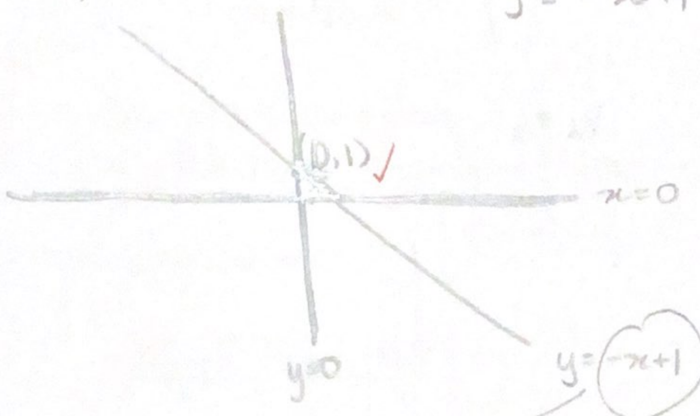
$(f_{xx})(f_{yy}) - (f_{xy})^2 = D$

$(2)(2) - 1 = 4 - 1 = 3 > 0 \Rightarrow D > 0$ and $f_{xx} > 0$

5

4. Find the volume under the plane $z = 1 - x - y$ over the domain D bounded by the lines $x = 0, y = 0$ and $x + y = 1$. in the first quadrant. (Note: this is the volume of a tetrahedron.)

$y = -x + 1$ ✓



$$\cancel{\int_0^{1}}\int_0^1 1-x-y \; dy\, dx$$

Set up as type 2
constants on the inside

$$\int_0^y \int_0^1 1-x-y \; dy\, dx$$

$$\int_0^y \left. y - xy - \frac{y^2}{2} \checkmark \right]_0^1 dx$$

$$\int_0^y \left[1 - x(1) - \frac{(1)^2}{2}\right] dx$$

$$= \int_0^y 1 - x - \frac{1}{2} \; dx = \int_0^y \left(\frac{1}{2} - x\right) dx$$

$$= \left. \frac{1}{2}x - \frac{x^2}{2} \right]_0^y \Rightarrow \frac{1}{2}y - \frac{y^2}{2}$$

$$\cancel{\int_0^x \int_0^1 1-x-y \; dx\, dy}$$

$$\cancel{\int_0^x \left. x - \frac{x^2}{2} - yx \right]_0^1 dy}$$

$$\cancel{\int_0^x 1 - \frac{1}{2} - y \; dy}$$

$$\cancel{\left. \frac{1}{2}y - \frac{y^2}{2} \right]_0^x}$$

$$\cancel{\frac{1}{2}x - \frac{x^2}{2}}$$

10

5. Convert to polar coordinates and then evaluate:

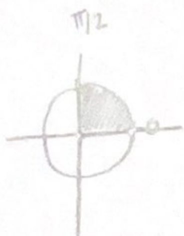
$$\int_0^1 \int_0^{\sqrt{1-x^2}} x^2 + y^2 \, dy \, dx.$$

$\sqrt{1-x^2} = y$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$

$$\int_0^{\pi/2} \int_0^1 x^2+y^2 \quad r\,dr\,d\theta$$



$$\int_0^{\pi/2} \int_0^1 \overset{r^2}{\cancel{(\quad\quad)}} \quad r\,dr\,d\theta$$

$$\int_0^{\pi/2} \int_0^1 r^2 \cdot r \quad dr\,d\theta$$

$$\int_0^{\pi/2} \int_0^1 r^3 \quad dr\,d\theta$$

$$\int_0^{\pi/2} \frac{r^4}{4}\Big]_0^1 \, d\theta$$

$$\int_0^{\pi/2} \left(\frac{1}{4} - \frac{0}{4}\right) d\theta$$

$$\int_0^{\pi/2} \frac{1}{4} \, d\theta$$

$$\frac{\theta}{4}\Big]_0^{\pi/2}$$

$$= \frac{\pi}{2} \div 4 = \cancel{\quad} - 0$$

$$= \frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$$