

MHF-4U TEST #1

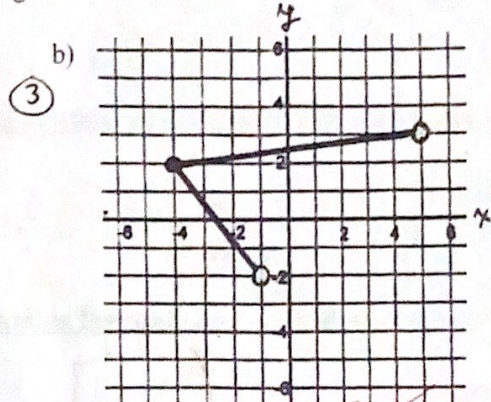
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68
76

Part A - Knowledge & Understanding (25)

1. For each relation state the domain, range and whether it is a function or not.

③ a) $f = \{ (1,2), (3,4), (5,2), (6,7) \}$
 function
 $D = \{1, 3, 5, 6\}$
 $R = \{2, 4, 7\}$



not a function
 $D = \{x \in \mathbb{R} \mid -4 \leq x < 5\}$
 $R = \{y \in \mathbb{R} \mid -2 < y < 3\}$

24
28
10
6

If there's a square root, it's not a function

③ c) $f(x) = \sqrt{x+4} - 10$
 Function
 $D = \{x \in \mathbb{R} \mid x \geq -4\}$
 $R = \{y \in \mathbb{R} \mid y \geq -10\}$

③ d) $y = \frac{7}{(x+3)(x-6)}$
 function
 $D = \{x \in \mathbb{R} \mid x \neq -3, 6\}$
 $R = \{y \in \mathbb{R} \mid y \neq 0\}$

2. For $f = \{ (4,7), (6,-3), (10,3), (12,7) \}$ and $g = \{ (6,5), (9,11), (10,5), (11,-6) \}$ find the following.

① a) $f - g$
 $(6, -8), (10, -2)$

① b) $f \times g$
 $(6, -15), (10, 15)$

2

14

2:

3. Evaluate, show steps. $3|7 - 13| - 5|13 - 9|$

(2)

$$\begin{aligned}
 &= 3|-6| - 5|4| \\
 &= 3(6) - 5(4) \\
 &= 18 - 20 \\
 &= -2
 \end{aligned}$$

4. a) Rewrite the following using absolute value symbol:

(1)

$$\begin{aligned}
 &\{x \in \mathbb{R} \mid -7 < x < 7\} \\
 &\{x \in \mathbb{R} \mid |x| < 7\}
 \end{aligned}$$

b) Rewrite the following using interval notation. $\{-5 < x \leq 4\}$

(1)

$$(-5, 4]$$

(\rightarrow greater than/not = to
 $[$ \rightarrow less than/= to

5. From the nine parent functions, that we studied in section 1.3 name:
(use equation to name function)

a) one that has two asymptotes and one that has one asymptote

(2)

$$\underline{y = \frac{1}{x}} \quad \underline{y = 2^x}$$

(1)

b) a function that oscillates $y = \sin x$

c) a function that increases throughout its entire domain ($x \in \mathbb{R}$) and does not have an asymptote and does not have a constant slope

(1)

$$\underline{y = x^3}$$

d) two functions where as $x \rightarrow -\infty, y \rightarrow \infty$ and as $x \rightarrow \infty, y \rightarrow \infty$

(2)

$$\underline{y = |x|} \quad \underline{y = x^2}$$

e) a function with an interval of increase $[0, \infty)$

(1)

$$\underline{y = \sqrt{x}}$$

Part B - Application (30)

1. Use algebra to prove the following odd, even or neither.

a) $f(x) = 5x^3 + \frac{2}{x}$

(2)

$$f(-x) = 5(-x)^3 + \frac{2}{(-x)}$$

$$= -5x^3 - \frac{2}{x}$$

$$= -(5x^3 + \frac{2}{x})$$

(2) 3

\therefore this function is odd because I got $-f(x)$ when I put in $f(-x)$

b) $f(x) = 7x^2 - 4|x|$

$$f(-x) = 7(-x)^2 - 4|-x|$$

$$= 7x^2 - 4|x|$$

\therefore this function is even because I got $f(x)$ when I put in $f(-x)$

you want to make it look the same

~~even if you have a - out and the left with f(x)~~

2. Fill in the appropriate numbers into the equation $y = \left(\frac{1}{2}\right)^x$ for the following transformations. All transformations in one equation.

- i) reflection in the x-axis ii) horizontal stretch by a factor of $\frac{1}{6}$ iii) vertical stretch by a factor of 10 iv) left 3 y) up 8

(5) 4

$$y = 10 \left(\frac{1}{2}\right)^{-6(x+3)} + 8$$

3. Find the inverse for $f(x) = -5(x - 8)^2 - 7$. Show work.

(3)

$$y = -5(x-8)^2 - 7$$

$$x = -5(y-8)^2 - 7$$

3

$$y = \pm \sqrt{\frac{x+7}{-5}} + 8$$

4.

③

4. The point (12, -8) is on the graph of $y = f(x)$. Find the corresponding point on the graph of $y = \frac{-1}{4} f(-2x - 14) + 3$. Show work.

$\hookrightarrow \frac{-1}{4} f(-2(x+7)) + 3$

$$x = -\frac{1}{2}x - 7 \rightarrow -\frac{1}{2}(12) - 7 = -13$$

$$y = \frac{-1}{4}y + 3 \rightarrow \frac{-1}{4}(-8) + 3 = 5$$

∴ the new point is (-13, 5)

5. Given the functions $f(x) = -3x^2 - 7x + 10$ and $g(x) = -4x + 5$ find the following

②

a) $f(g(2))$

$$g(2) = -4(2) + 5 = -8 + 5 = -3$$

①

$$f(-3) = -3(-3)^2 - 7(-3) + 10 = -27 + 21 + 10 = 4$$

②

b) $g^{-1}(g(18))$

$$g(18) = -4(18) + 5 = -72 + 5 = -67$$

$$g^{-1}(-67)$$

Inverse equation $\rightarrow g^{-1}(x) = \frac{x-5}{-4}$

$$= \frac{-67-5}{-4} = \frac{-72}{-4} = 18$$

c) $h(x) = f(x) \times g(x)$

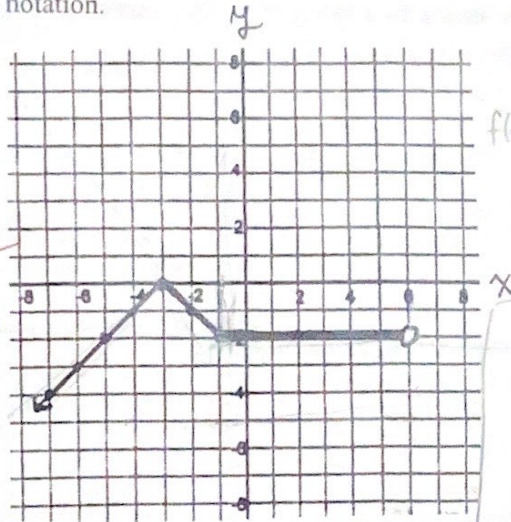
$$(-3x^2 - 7x + 10)(-4x + 5)$$

$$= 12x^3 - 15x^2 + 28x^2 - 35x - 40x + 50$$

$$= 12x^3 + 13x^2 - 75x + 50$$

6. Write the algebraic representation for the following piecewise function, using function notation.

④



$$f(x) = \begin{cases} |x+3|, & \text{if } x \leq -1 \\ -2, & \text{if } -1 < x \leq 6 \end{cases}$$

- |x|
can put \leq on both

$$70xy - 84x - 35y + 42$$

$$= 14x(5y-6) - 7(5y-6)$$

$$= (14x-7)(5y-6)$$

$$= 7(2x-1)(5y-6)$$

7. Factor completely. Show work

③

a) $30x - 25 + 49y^2 - 9x^2$

$$= -9x^2 + 30x - 25 + 49y^2$$

$$= -9x^2 + 15x + 15x - 25 + 49y^2$$

$$= -3x(3x-5) + 5(3x-5) + 49y^2$$

$$= (3x-5)(-3x+5) + 49y^2$$

$$= -(3x-5)^2 + 49y^2$$

③

b) $70xy - 84x - 35y + 42$

$$= -((3x-5)-7y)((3x-5)+7y)$$

$$= -(3x-5-7y)(3x-5+7y)$$

*

Part C - Communication (12)

5

1. Describe the transformations, in words, that would be needed to get the graph $y = \frac{1}{5} (2^{-3(x-4)}) - 9$, from the parent graph $y = 2^x$. (one per line)

- Vertical compression by a factor of $\frac{1}{5}$ (multiply y-values by $\frac{1}{5}$)

- Horizontal compression by a factor of $\frac{1}{3}$ (multiply x-values by $\frac{1}{3}$)

- reflection in x-axis (switch signs on x-values)

- Horizontal shift 4 units right (add 4 to x-values)

- Vertical shift 9 units down (subtract 9 from y-values)

2. Describe the difference in symmetry between an odd function and an even function.

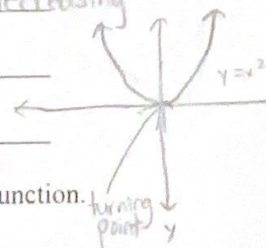
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An even function is symmetrical about the y-axis, such as $f(x) = |x|$ or $f(x) = x^2$. An odd function has rotational symmetry by 180° , such as x^3 .

3. Describe what a turning point is on a curve (graph).

1

It is a point where a graph changes from increasing to decreasing or decreasing to increasing, such as $f(x) = x^2$



4. Discuss the vertical and horizontal continuity/discontinuity for the piecewise function. Use specific values.

$f(x) = 5x - 21$ for $0 \leq x < 6$ and $f(x) = -4x + 34$ for $6 \leq x < 10$

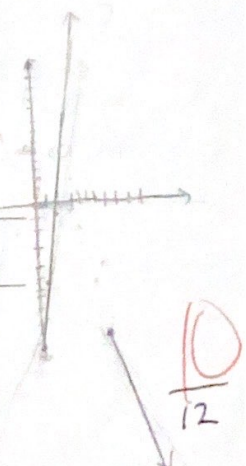
$f(6) = 5(6) - 21 = 9$

$f(6) = -4(6) + 34 = 10$

horizontal → at $x=6$, the graphs are discontinuous. They are not continuous (I plugged in 6 for the equations and got different answers) even though the x's are all covered from 0 to 9.99.

vertical → They are vertically discontinuous as well

there is horizontal continuity b/c all the x's are filled in.



10/12

6.

Part D - Thinking & Inquiry (9)

④ 1. Find the inverse of $f(x) = 1 - \sqrt{\frac{x+5}{4}}$.

~~$y = 1 - \sqrt{\frac{y+5}{4}} + 1$~~

$\left(\frac{y-1}{-1}\right)^2 = \left(\sqrt{\frac{x+5}{4}}\right)^2$

$4(-y+1)^2 - 5 = x$

$y = 4(x-1)^2 - 5$

$y = 4(-x+1)^2 - 5$

$\therefore y = -4(x-1)^2 - 5, x \leq 1 \rightarrow$ one branch only

2. The point (3, 6) is on the graph of $y = 2f(x+1) - 4$. Find the original point on the graph of $y = f(x)$ that was transformed to (3, 6). Show work.

③

$x = x - 1$

$y = 2y - 4$

$3 = x - 1$

$6 = 2y - 4$

$x = 4$

$10 = 2y$

$y = 5$

\therefore the original coordinates were (4, 5)

To check, sub points back in

$x = 4 - 1$

$y = 2(5) - 4$

$x = 3$

$y = 6$

②

3. State the domain and range for $f(x) = \frac{2}{x^2} - 8$.

$D = \{x \in \mathbb{R} \mid x \neq 0\}$

$R = \{y \in \mathbb{R} \mid y \geq -8\}$

6/9