83%

1.

$\frac{35}{42}$

11
10
9
5

MHF-4U **TEST#2** (Chapter 2) [redacted]

Part A – Knowledge & Understanding (12)

1. The following table shows the temperature of an oven as it heats up.

Time (min)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Temperature (°F)	70	125	170	210	250	280	310	335	360	380	400	415	430	440	445

(1) a) What is room temperature in the kitchen? 70° .

(2) b) Find the average rate of change for the full 14 minutes. (one decimal)

2

$0 \le t \le 14$ $\frac{y_2 - y_1}{x_2 - x_1}$ $(0, 70)$ $(14, 445)$ $\frac{445-70}{14-0} = \frac{375}{14} = 26.8°/\text{minute}$

(2) c) Find the average rate of change for 8 to 11 minutes. (one decimal)

2

$8 \le t \le 11$ $(8, 360)$ $(11, 415)$ $\frac{415-360}{11-8} = \frac{55}{3} = 18.3°/\text{minute}$

(2) d) Find the average rate of change for 11 to 14 minutes. (one decimal)

2

$11 \le t \le 14$ $(11, 415)$ $(14, 445)$ $\frac{445-415}{14-11} = \frac{30}{3}$ → $10°/\text{minute}$

(1) e) Use your answers in c) and d) to find the instantaneous rate of change at 11 mins. (1 decimal)

$\frac{18.3+10}{2} = 14.1°/\text{minute}$

$= 14.15$ → 14.2

(2) f) Use the most accurate **centered interval** available to find IROC at 11 mins. (1 decimal)

2

$10 \le t \le 12$ $(10, 400)$ $(12, 430)$ $\frac{430-400}{12-10} = \frac{30}{2}$ → $= 15°/\text{minute}$

g) Explain the difference in your answers for the IROC at 11 mins. in e) and f).

(2) 2 In e) I found the IRoC by using a following and preceding interval and dividing by 2 (finding their average) In f) I found the IRoC by a centered interval, which is more accurate

$\frac{11}{12}$

2.

$(-1.01)^3 - 3(-1.01)$

$= -1.030301 + 3.03$

$= 1.999699$

$-0.970299 + 2.97$

Part B – Application (12)

1. Given $f(x) = x^3 - 3x$ and the point $(-1, 2)$.

a) Use the difference quotient and $h = -0.01$ (preceding interval) to find the IROC at $x = -1$.

(3) 2

$\frac{f(a+h)-f(a)}{h}$ $\rightarrow \frac{f(-1.01)-f(-1)}{-0.01} = \frac{1.999699-2}{-0.01} = 0.0301$

$h = -0.01$

$a = -1$

b) Use the difference quotient and $h = 0.01$ (following interval) to find the IROC at $x = -1$.

(3) 3

$\frac{f(a+h)-f(a)}{h}$

$\frac{f(-1+0.01)-f(-1)}{0.01}$ $\rightarrow \frac{f(-0.99)-f(-1)}{0.01} = \frac{1.999701-2}{0.01}$

$= -0.0299$

c) Use your answers in a) and b) to find a more accurate value for the IROC at $x = -1$.

(1)

$\frac{0.0301+(-0.0299)}{2} = 0.0001 = 0$

d) Explain whether $(-1, 2)$ is a maximum or a minimum point using a), b) and c).

(2)

The slope of the preceding interval is positive from a)

The slope of the following interval is negative from b)

∴ It is at a maximum at (-1,2)

Use answer in c) → IROC=0

2. Given $k(x) = -x^4 + 8x^3 - 20x^2 + 16x$, use $h = 0.001$ to find the IROC at $(2, 0)$.

(3) 2

$\frac{f(a+h)-f(a)}{h}$

$a = 2$

$h = 0.001$

$\frac{f(2.001)-f(2)}{0.001}$

$\rightarrow = -(2.001)^4 + 8(2.001)^3 - 20(2.001)^2 + 16(2.001)$

$= 48.064024 - 80.08002 + 32.016$

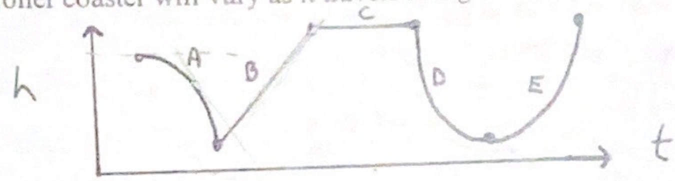
$= 0.000004$

$\frac{\quad}{0.001} = 0.004$

$\frac{10}{12}$

Part C – Communication (10)

1. The graph shows the height of a roller coaster versus time. Describe how the **vertical** speed of the roller coaster will vary as it travels along the track from A to E.



(5)

A - Starts ~~At~~ at a maximum, goes at an increasing speed towards the ground

B - goes at a constant speed back up

C - Stays at a constant speed = 0

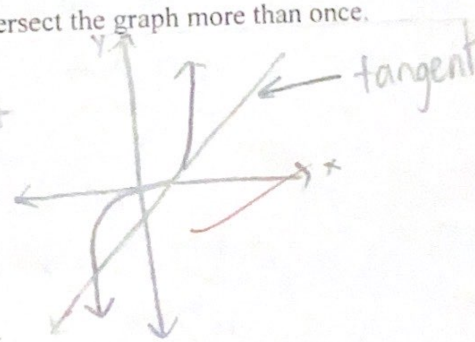
D - goes at a decreasing speed closer to the ground

E - goes ~~at~~ on increasing speed back to maximum height

2. Given a **cubic** function, is it possible for a tangent to intersect the graph more than once. (give a diagram)

(2)

Yes, because a tangent is any line that goes through a graph once over a given interval, but may cross at other points outside that interval



3. Describe how you would solve the following word problem. **DO NOT SOLVE.**

(3)

The movement of a certain glacier can be modelled by $d(t) = 0.01t^2 + 0.5t$, where d is the distance, in metres, that a stake on the glacier has moved, relative to a fixed position, t days after the first measurement was made. Estimate the rate at which the glacier is moving after 20 days.

Find IROC.

you could use a centered interval so your answer will be more accurate. $19 \le t \le 21$.

Sub d(19) and d(21) in $d(t) = 0.01t^2 + 0.5$ and find the y values when d=19 and when d=21. Then you have 2 pairs of coordinates. Find the slope between them $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$ and you have the IROC.

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4.

8.01200601 - 8.012

Part D – Thinking & Inquiry (8)

1. Show that the minimum or maximum value for the function $f(x) = x^3 - 12x + 16$ happens at $x = 2$. Use the **difference quotient** and an **algebraic solution** then simplify before using $h = 0.01$ and $h = -0.01$ to justify your solution. SHOW ALL WORK.

8

$$\frac{f(a+h)-f(a)}{h}$$

$$= \frac{f(2+h)-f(2)}{h}$$

$(2+h)^3 - 12(2+h) + 16$
$= (2+h)(4+h^2+4h) - 24 - 12h + 16$
$= 8 + 2h^2 + 8h + 4h + h^3 + 4h^2 - 24 - 12h + 16$
$= h^3 + 6h^2$

$(2)^3 - 12(2) + 16$
$= 8 - 24 + 16$
$= 0$

$$= \frac{h^3 + 6h^2 - 0}{h}$$

$$= \frac{h^3 + 6h^2}{h}$$

$$= \frac{h^2(h+6)}{h}$$

$= h(h+6)$

$= h^2 + 6h$

Slope should be zero at the max or min:

following
$h = 0.001$
$(0.001)^2 + 6(0.001)$
$= 0.006001$

preceding
$h = -0.001$
$(-0.001)^2 + 6(-0.001)$
$= 0.000001 - 0.006$
$= -0.005999$
$= 0.000001$
$= 0$

$$\frac{f(a+h)-f(a)}{h}$$

$$\frac{f(1.999)-f(2)}{-0.001}$$

$= -0.005999$

$$\frac{f(2.001)-f(2)}{0.001}$$

$= 0.006001$

$$IROC = \frac{0.0601 + (-0.0599)}{2}$$

$= 0.0001$

≈ 0

(2,0) is a minimum val

The slope of the preceding interval is negative, slope of the following interval is positive

∴, it's at a minimum.

5/8