

83%  
60  
72

1. Solve algebraically and check one solution. Show all work.

7)

When there's an = sign you can get rid of the denominator

$$\frac{-2}{x-4} = \frac{x+11}{x+4}$$

$$\begin{aligned} -2(x+4) &= (x+11)(x-4) \\ -2x-8 &= x^2-44-4x+11x \\ -2x-x^2+4x-11x &= -44+8 \\ -x^2-9x+44-8 &= 0 \\ -x^2-9x+36 &= 0 \\ -(x^2+9x-36) &= 0 \\ -(x+12)(x-3) &= 0 \\ x+12=0 & \quad x-3=0 \\ x=-12 & \quad x=3 \end{aligned}$$

∴, x = -12 and x = 3

check: LS	RS
$\frac{-2}{x-4}$	$\frac{x+11}{x+4}$
$= \frac{-2}{(3)-4}$	$= \frac{3+11}{3+4}$
$= \frac{-2}{-1}$	$= \frac{14}{7}$
$= 2$	$= 2$

LS = RS  
∴, x = 3

24  
22  
6  
8

2. If the demand/price function for a company is  $p(x) = \frac{5}{2x^2 - 3x - 7}$  and x is the number of sales in thousands, find:

a) The revenue (R(x)) function for this company.

$$R(x) = x \cdot p(x) = x \cdot \frac{5}{2x^2 - 3x - 7} = \frac{5x}{2x^2 - 3x - 7}$$

b) The average rate of change in revenue between 2000 and 6000 sales. Final answer 2 decimals.

x = 2000

y = 5(2000) = 10000

$$\frac{2(2000)^2 - 3(2000) - 7}{8000000 - 6000 - 7} = \frac{8000000 - 6000 - 7}{8000000 - 6000 - 7}$$

~~x = 6000~~

x = 6000

y = 30000

$$\frac{72000000 - 18000 - 7}{72000000 - 18000 - 7} = 0.00041677$$

~~(2000, 0.001250939)~~

~~(6000, 0.00041677)~~

$$\frac{y_2 - y_1}{x_2 - x_1} = \text{AROC}$$

~~(420, 2, -2)~~

~~(6, 30, 6382)~~

c) The marginal revenue for 4000 sales. Final answer 2 decimals. (h = 0.001)

$$\frac{f(a+h) - f(a)}{h} = \frac{1.537308609 - 0.327861}{0.001} = 1209.44$$

h = 0.001

a = 4000

f(4000.001) - f(4000)

0.001

f(4.001) = 1.537308609 = 0  
f(4) = 0.327861

= -1.15

AROC = 0.6382 - (-2)

AROC = 0.65

0.000075234 - 0.0002523

0.001

= 0

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2:

3. Solve using algebra and a factor table. Show all work. Final answers in interval notation.

$$\frac{x^2+3x}{x-2} \geq \frac{x^2+10x}{x+4}$$

$$\frac{x^2+3x}{x-2} - \frac{x^2+10x}{x+4} \geq 0$$

$$\frac{(x^2+3x)(x+4) - (x^2+10x)(x-2)}{(x-2)(x+4)} \geq 0$$

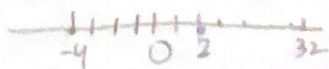
$$\frac{x^3+3x^2+4x^2+12x - (x^3+10x^2-20x-2x^2)}{(x-2)(x+4)} \geq 0$$

$$\frac{x^3+7x^2+12x - x^3-10x^2+20x+2x^2}{(x-2)(x+4)} \geq 0$$

$$\frac{-x^2+32x}{(x-2)(x+4)} \geq 0$$

$$\frac{-x(x-32)}{(x-2)(x+4)} \geq 0$$

$-x=0$   
 $x-2=0 \rightarrow x=2$   
 $x+4=0 \rightarrow x=-4$   
 $x-32=0 \rightarrow x=32$



$x < -4$   
 $-4 < x < 0$   
 $0 < x < 2$   
 $2 < x < 32$   
 $x > 32$

	$x < -4$	$-4 < x < 0$	$0 < x < 2$	$2 < x < 32$	$x > 32$
$x$	-	-	+	+	+
$(x-2)$	-	-	-	+	+
$(x+4)$	-	+	+	+	+
$(x-32)$	-	-	-	-	+
LC	-	-	-	-	-
$\frac{-x(x-32)}{(x-2)(x+4)}$	-	+	-	+	-

$x \neq 2, -4$

$-4 < x < 0$   
 $[-4, 0]$

$2 < x < 32$   
 $(2, 32]$

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Part B - Application (25)

1. Sketch a graph, on the graph paper provided, of  $f(x) = -0.5x^2 - 2x + 2.5$ , including all major points. Show work. Use this graph to sketch a graph of the reciprocal function

$f(x) = \frac{1}{-0.5x^2 - 2x + 2.5}$ . Show work to get all major points, asymptotes and label on the graph.

$$f(x) = -0.5x^2 - 2x + 2.5$$

$$= -0.5(x^2 + 4x - 5)$$

$$= -0.5(x-1)(x+5)$$

X Int.

$$\begin{cases} x-1=0 & x+5=0 \\ x=1 & x=-5 \end{cases}$$

when  $y=1$

$$1 = -0.5x^2 - 2x + 2.5$$

$$0 = -0.5x^2 - 2x + 1.5$$

$$\begin{aligned} a &= -0.5 \\ b &= -2 \\ c &= 1.5 \end{aligned}$$

$$\frac{2 \pm \sqrt{4 - 4(-0.5)(1.5)}}{2(-0.5)}$$

$$= \frac{2 \pm 2.6457}{-1}$$

$$\begin{aligned} & \swarrow \quad \searrow \\ & -4.6457 \quad 0.6457 \end{aligned}$$

Vertex:

$$\frac{-b}{2a}, \frac{4ac-b^2}{4a} \quad \begin{aligned} a &= -0.5 \\ b &= -2 \\ c &= 2.5 \end{aligned}$$

$$\frac{-(-2)}{2(-0.5)}, \frac{4(-0.5)(2.5) - (-2)^2}{4(-0.5)}$$

$$= (-2, 4.5)$$

When  $y=-1$

$$-1 = -0.5x^2 - 2x + 2.5$$

$$0 = -0.5x^2 - 2x + 3.5$$

$$a = -0.5 \quad b = -2 \quad c = 3.5$$

$$\frac{-(-2) \pm \sqrt{4 - 4(-0.5)(3.5)}}{2(-0.5)}$$

$$= \frac{2 \pm 3.3166}{-1}$$

$$\begin{aligned} & \swarrow \quad \searrow \\ & -5.3166 \quad 1.3166 \end{aligned}$$

Reciprocal graph

$$\frac{1}{-0.5(x-1)(x+5)} \quad \begin{aligned} & \nearrow x \neq 1, -5 \\ & \downarrow \end{aligned}$$

Vertex of reciprocal:  $(-2, \frac{1}{4.5})$

Points:  $(-4.6457, 1), (0.6457, 1), (-5.3166, -1), (1.3166, -1)$  V.A.

Vertical asymptotes at  $x = 1, -5$

Horizontal asymptote at  $y = 0$

Y Int:  $\frac{1}{2.5}$   
 $\hookrightarrow (0, 0.4)$

4.

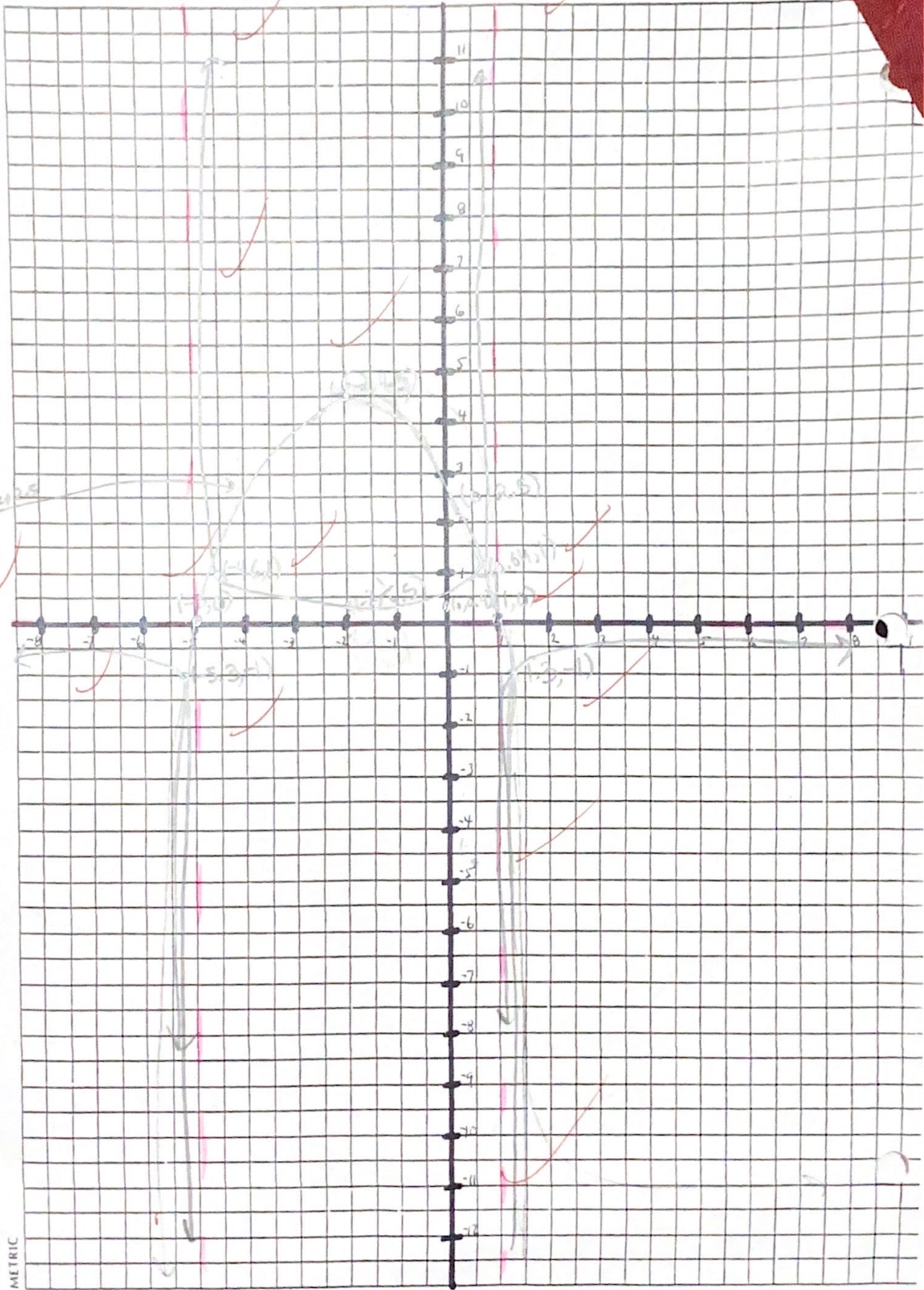
$x = -5$

y

$x = 1$

$y = -0.5x^2 + 2x + 2.5$

$y = 0$

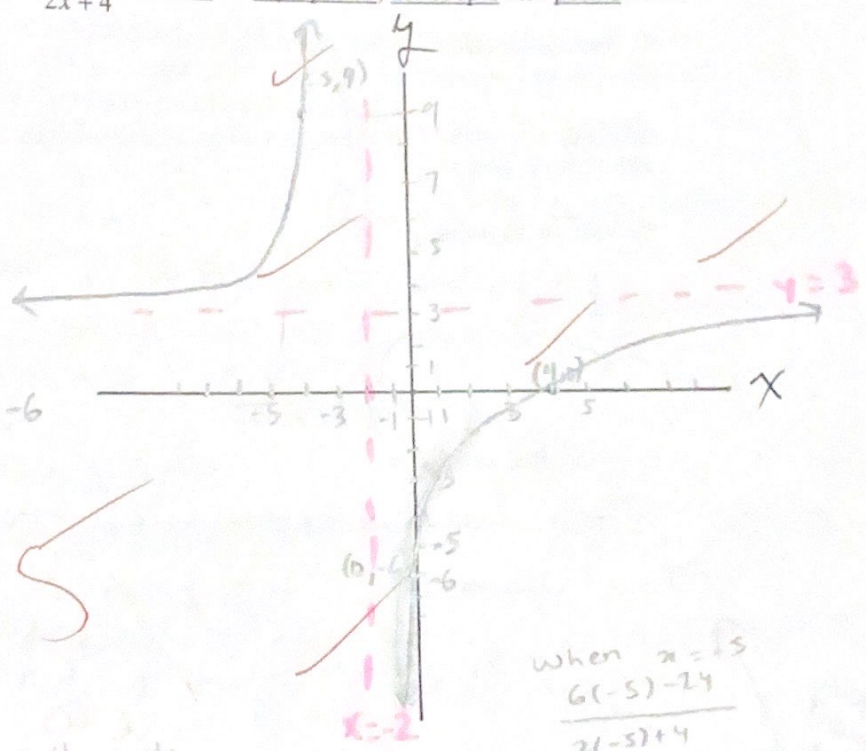


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METRIC

2. Sketch the graph of  $f(x) = \frac{6x-24}{2x+4}$ , Label all asymptotes, intercepts and points used. Show work.

$\frac{a}{c} = h.a. = \frac{6}{2}$   
 Horizontal asymptote at  $y=3$   
 Vertical asymptote at  $x=-2$   
 y Int:  $y = \frac{6(0)-24}{2(0)+4} = \frac{-24}{4} = -6$   
 $\hookrightarrow (0, -6)$   
 x Int:  $\frac{6(x-4)}{2(x+2)}$



$x-4=0 \Rightarrow x=4$   
 $x+2=0 \Rightarrow x=-2$  doesn't work, it's an asymptote

when  $x = -5$   
 $\frac{6(-5)-24}{2(-5)+4} = \frac{-30-24}{-10+4} = \frac{-54}{-6} = 9$

3. Alana takes a certain amount of time to clean and wash a car. Brad takes 12 minutes more than Alana and together they take 20 minutes. How long does it take each person to wash and clean a single car on their own? Use algebraic solution with final answer to 2 decimals.

∴ Alana takes 20 mins on her own, and Brad takes (20+12) 32 mins to clean the car on his own

Alana =  $x$  Let  $x$  = the time Alana takes  
 Alana takes  $\frac{1}{x}$  amount of time  
 Brad takes  $\frac{1}{x+12}$

Together:  $\frac{1}{x} + \frac{1}{x+12} = \frac{1}{20}$   
 $\frac{1(x+12) + 1(x)}{x(x+12)} = \frac{1}{20}$

$\frac{x+12+x}{x^2+12x} = \frac{1}{20}$   
 $\frac{2x+12}{x^2+12x} - \frac{20(x^2+12x)}{x^2+12x} = 0$

$0 = \frac{2x+12-20x^2-240x}{x^2+12x}$   
 $\frac{-20x^2-238x+12}{x^2+12x}$   
 $a = -20 \quad b = -238 \quad c = 12$   
 $238 \pm \sqrt{56644 - 4(-20)(12)}$   
 $\frac{238 \pm 240.008}{-40}$   
 $\frac{238 + 240.008}{-40} = -11.9502$  (Not possible)  
 $\frac{238 - 240.008}{-40} = 0.0502$   
 $\frac{1}{0.0502} = 19.92 \text{ min}$  (20)  
 $\frac{1}{-11.9502} = -0.0837$  (Not possible)

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6.

Part C - Communication (16)

1. For the function  $g(x) = \frac{x-5}{x^2-2x-15}$

Explain why each of the following characteristics is present or not and find the appropriate equation or point.

(2) a) Vertical asymptote

Yes, because there are values that x can equal and that makes the equation = 0, which can't happen.  $\frac{(x-5)}{(x-5)(x+3)}$  Vertical asymptote at  $x = -3$

(2) b) Horizontal asymptote

~~HA at  $y = 0$  because the equation will = 0~~

No, because when you cross the (x-5) factors you get  $y = \frac{1}{x+3}$ , and there is no 'a' value to find a HA. With  $\frac{a}{a}$   $a = 0$  Any linear equation has HA @  $y = 0$

(2) c) Hole

Yes, you can cross the (x-5) factors, resulting in  $\frac{1}{x+3}$ . When  $x = 5$ ,  $\frac{1}{5+3} = \frac{1}{8} \rightarrow y = \frac{1}{8}$ . There is a hole at  $(5, \frac{1}{8})$ .

(2) d) Slant or Oblique asymptote

No the degree of the top is not greater than the degree of the bottom by 1. The degree of the top is 1 and the bottom is 2.

You won't have a t HA or OA at once

2. Describe the solution for  $f(x) = \frac{(x-2)(x+7)}{x+7}$ . Cancel!

(2) hole @  $x = -7$   
 $y = x - 2$   
 $x^2 + 5x - 14$   
 $(x+7)(x-2)$   
 $(-7, -9)$

$x \neq -7$ , asymptote at  $x = -7$   
The degree of the top is greater than the bottom so there is an oblique asymptote at  $x - 2$

No H.A.  
Zeroes:  $x = -7$

$-7 \begin{array}{r|rrr} 1 & +5 & -14 & \\ & -7 & & 14 \\ \hline & 1 & -2 & 0 \end{array}$

(1)  
6  
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Part D - Thinking & Inquiry (11)

1. Brad bought some video games for \$900. He kept 3 games for himself and sold the rest for \$1020, making a profit of \$15 on each game. How many games did he buy originally? Set up an appropriate equation and give an algebraic solution.

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$$\frac{1020}{x-3} - \frac{900}{x} = 15$$

Let  $x$  = the number of games bought originally

Common denominator:  $x(x-3)$

$$x(x-3) \left( \frac{1020}{x-3} - \frac{900}{x} \right) = 15x(x-3)$$

$$\frac{x(x-3)(1020) - 900x(x-3)}{(x-3)(x)} = 15$$

$$\frac{1020x(x-3) - 900x(x-3)}{(x-3)(x)} = 15$$

Multiply both sides by  $x(x-3)$   
Bring everything to one side after cancelling denominators  
and factor from quadratic formula and get the 2 zeroes

$$1020x^2 - 3060x - 900x^2 + 2700 = 15x^2 - 45x$$

$$120x^2 - 15x + 45 - 3060x + 2700 = 0$$

2. Make up a rational equation that has a vertical asymptote at  $x = -4$  and an oblique asymptote at  $y = x - 3$ .

3

$$\frac{x^2 + x + 1}{x + 4}$$

$$-4 \begin{array}{r|l} 1 & -4 \\ \hline 1 & -3 \end{array}$$

$$x + 4 = 0$$

$$x \neq -4$$

Divide to find oblique asymptote

$$-4 \begin{array}{r|ll} 1 & -4 & 1 \\ \hline 1 & -3 & 13 \end{array}$$

$x - 3$

~~$105x^2 - 3015x + 2700$~~   
 ~~$3015x - 2820.67$~~   
 ~~$210$~~   
 ~~$27.78$~~   
 ~~$0.92$~~

He bought 28 games