81%

MHF-4U **TEST # 6** (chapter 6) [redacted]

$\frac{55}{68}$

Part A – Knowledge & Understanding (22)

(2) 1. Convert to radians. a) using π and fraction b) to 3 decimals.

a) 255^0 $225 \times \frac{\pi}{180} = \frac{5\pi}{4}$ or 3.926 rad

b) -328^0 $-328 \times \frac{\pi}{180}$ $= -5.724$ or $\frac{-82\pi}{45}$ rad

2

2. Convert to degrees. **One** decimal.

(2) a) $\frac{-11\pi}{14}$ rad $\frac{-11\pi}{14} \times \frac{180}{\pi} = -141.4°$

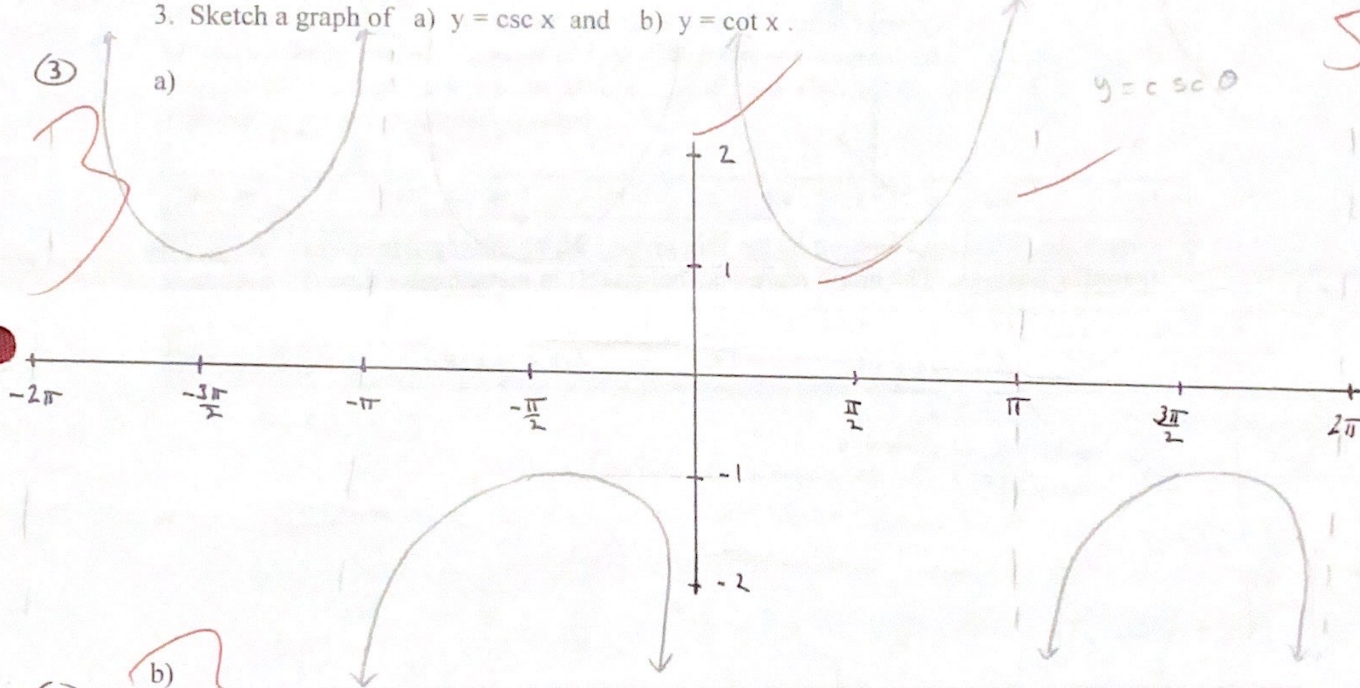
b) 8.2 rad $\frac{8.2}{1} \times \frac{180}{\pi} = \frac{1476}{\pi} = 469.8°$

18 25 7 5

3. Sketch a graph of a) y = csc x and b) y = cot x .

(3) a)

3



(3) b)

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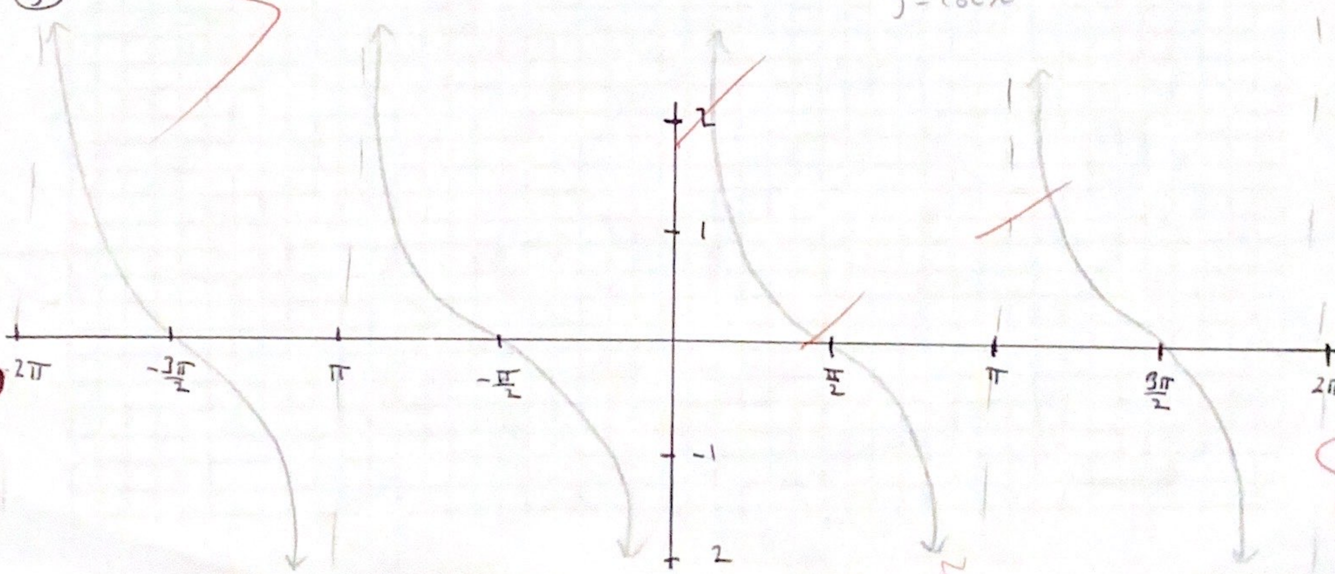

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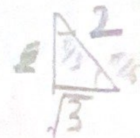
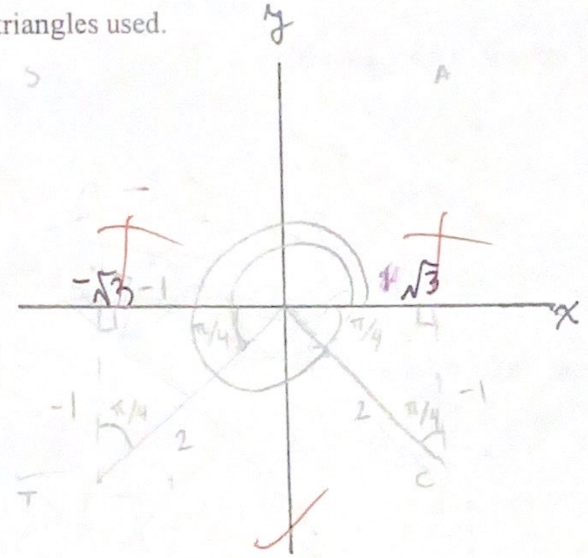
2.

5

4. Use **special triangles**, a **complete sketch** from $\{ 0 \leq \theta \leq 2\pi \}$ to find θ in radians in fractional form (with π) for $\sin \theta = -\frac{1}{2}$. A complete diagram includes all 3 sides of any triangles used.





$\text{Sin} = \frac{y}{r} = -\frac{1}{2}$

$\text{Sin}\,\theta = -\frac{1}{2} \rightarrow$ Sin is negative in the 3rd and 4th quadrants

$x = \pm 1$

$y = -1$

$r = 2$

$\theta_1 = \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$

$\theta_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

5. Graph the following equation on the grid given. Set up appropriate scales and **show all transformations**. The graph represents height (H) in metres, after time (t) in seconds.

7

$$H(t) = -12 \cos\left(\frac{\pi}{2}(t+1)\right) + 14 \qquad \{ 0 \leq t \leq 8 \}$$

$a = 12$

reflection in x-axis

moved left 1 (horizontal translation)

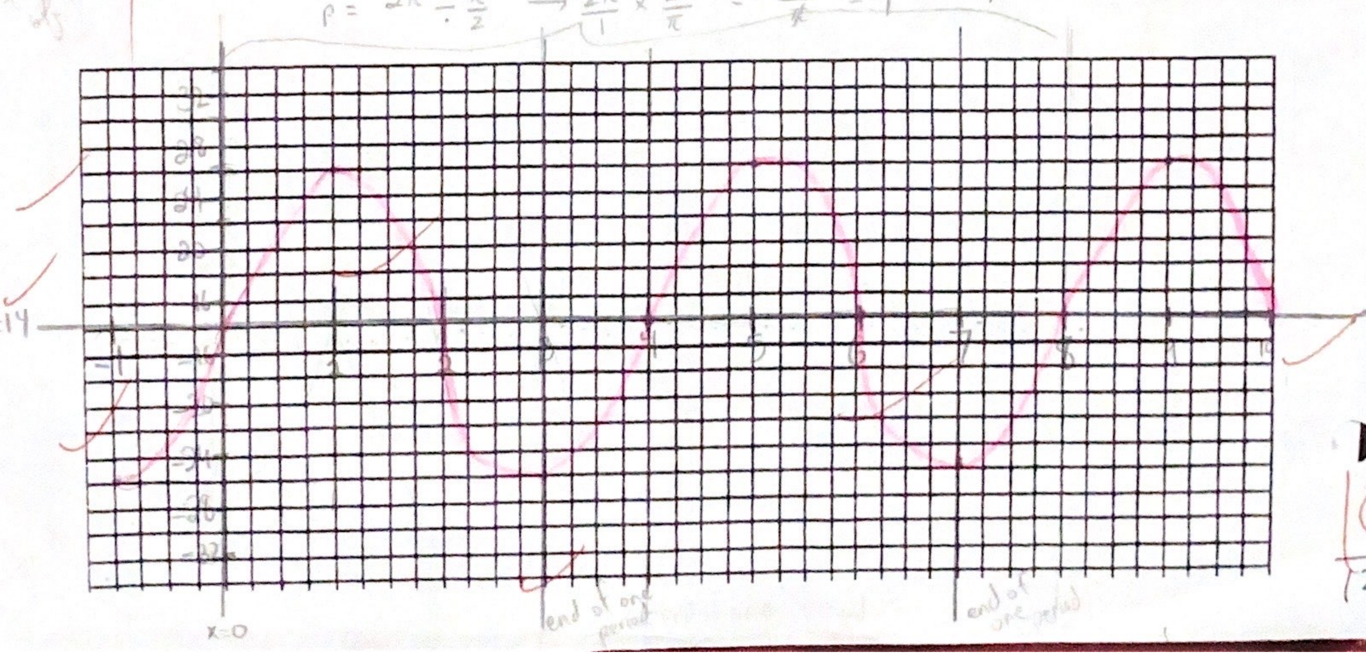
up 14 (vertical translation)

period $= \frac{2\pi}{k}$

$p = 2\pi \div \frac{\pi}{2} \rightarrow \frac{2\pi}{1} \times \frac{2}{\pi} = \frac{4\pi}{\pi} = 4$

$0 \leq t \leq 8$

show 1 graph of transformations?



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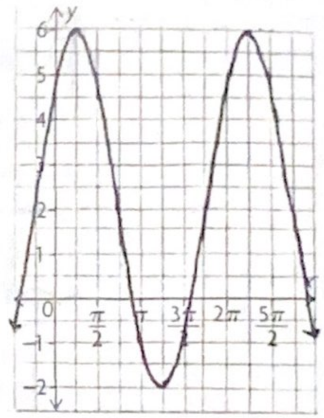
Part B – Application (26)

1. Find the equation of the graph using a) y = cos x and b) y = sin x .



(4)

a) $y = \cos x$, period $= 2\pi$; max $= 6$, min $= -2$; eoa $= \frac{6+(-2)}{2} = 2$; amplitude $= \frac{6-(-2)}{2} = 4$

moved $\frac{\pi}{4}$ right $\rightarrow$ $\boxed{y = 4\cos\left(x - \frac{\pi}{4}\right) + 2}$

4

(2)

b) $y = \sin x$; eoa $= 2$; amp. $= 4$; $P = 2\pi$

moved left $\frac{\pi}{4}$ $\rightarrow$ $\boxed{y = 4\sin\left(x + \frac{\pi}{4}\right) + 2}$

2

(4)

2 Find the trig. equation for the data given in the table.

Melissa used a motion detector to measure the horizontal distance between her and a child on a swing. She stood in front of the child and recorded the distance, $d(t)$, in metres over a period of time, t, in seconds. The data she collected are given in the following tables

Time (s)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
Distance (m)	3.8	3.68	3.33	2.81	2.2	1.59	1.07	0.72	0.6	0.72	1.07	1.59

Time (s)	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
Distance (m)	2.2	2.81	3.33	3.68	3.8	3.68	3.33	2.81	2.2	1.59	1.07	0.72	0.6

max $= 3.8$
min $= 0.6$
Started at a maximum $\rightarrow$ use cos

eoa $= \frac{3.8+0.6}{2} = 2.2$

amp. $= \frac{3.8-0.6}{2} = 1.6$

period $= 1.6$

$k = \frac{2\pi}{P} = \frac{2\pi}{1.6} \doteq 3.92 = \frac{5\pi}{4}$

$y = 1.6\cos\left(\frac{5\pi}{4}x\right) + 2.2$

4

3. Determine the exact value using radicals (if needed) and fractions for the following. Sketch an appropriate triangle for each angle in standard position.

(2)

a) $\cot \frac{5\pi}{3} = \frac{x}{y} = \frac{1}{-\sqrt{3}}$ — don't put negatives on bottom of fractions

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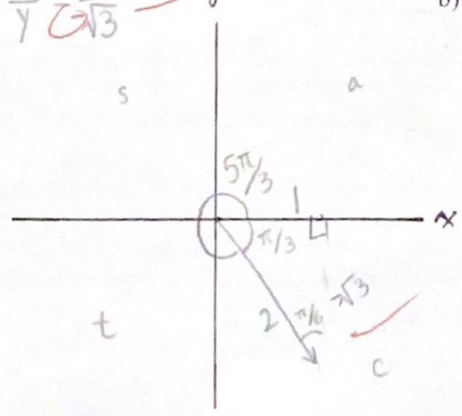
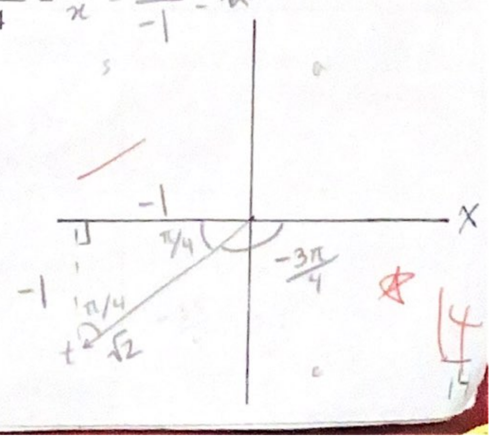


(2)

b) $\sec \frac{-3\pi}{4} = \frac{r}{x} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$



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4.

4. The daily high temperature of a city, in degrees Celsius, as a function of the number of days into the year, can be described by the function $T(d) = -17\cos\left(\frac{2\pi}{365}(d-8)\right) + 22$.

(3) a) What is the average rate of change of the daily temperature from day 50 to day 110? Show work.

$ARoC = \frac{y_2 - y_1}{x_2 - x_1}$

3

$x = 50$

$y = -17\cos\left(\frac{2\pi}{365}(42)\right) + 22$

$= 9.25295118$

$x = 110$

$y = -17\cos\left(\frac{2\pi}{365}(102)\right) + 22$

$= 25.12797198$

$(50, 9.25295118)\ y_1$

$(110, 25.12797198)\ y_2$

$\frac{25.12797198 - 9.25295118}{110 - 50}$

$ARoC = 0.2645\ °/day$

(3) b) What is the IROC on day 325. Show work. Use h = 0.01.

3

$a = 325$

$h = 0.01$

$\frac{f(a+h) - f(a)}{h}$

$\frac{f(325.01) - f(325)}{0.01}$

$f(325.01) = -17\cos\left(\frac{2\pi}{365}(317.01)\right) + 22$

$= 10.47839662$

$f(325) = -17\cos\left(\frac{2\pi}{365}(317)\right) + 22$

$= 10.48054859$

$IRoC = \frac{10.47839662 - 10.48054859}{0.01}$

$= -0.215197$?

$-0.22\ °/day$

-1

(6) 5. Given $\tan\theta = -\frac{24}{7}$, find all values for θ in radians (2 decimals) for $\{0 \le \theta \le 6.28\}$.

Sketch appropriate triangle(s) for θ and label diagram for x, y, **and r**

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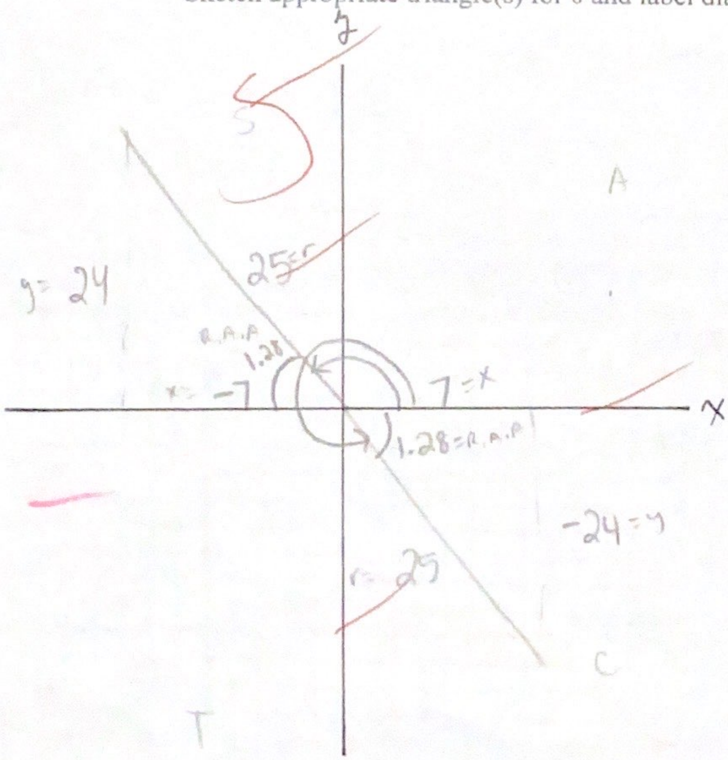

$\tan = \frac{y}{x} = \frac{-24}{7}$

$\tan^{-1}\left(\frac{-24}{7}\right)$

$= -1.28 \rightarrow$ R.A.A. is 1.28

$+6.28$

$\theta_1 = 5$ rad

$24^2 + 7^2 = r^2$

$r = 25$

$\theta_2 = \pi - 1.28$

$\theta_2 = 1.86$ rad

$\frac{11}{12}$

5.

Part C – Communication (8)

1. Describe, **in general**, in words, where you would find the following situations in the given graph.

a) **sinusoidal graph (y = sin x or y = cos x)**

i) IROC = 0 at any max or min point

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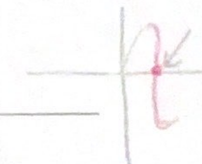
ii) AROC = 0 any horizontal line between 2 points on the graph

iii) IROC is a **minimum** (lowest possible negative value)



right between a max and a min value

4

iv) AROC > 0 (positive)

when slope is positive, from any lower to high point (on y-values), when graph is increasing

4

2. A bumblebee is flying in a circular motion within a vertical plane, at a constant speed. The height of the bumblebee above the ground, as a function of time, can be modelled by a sinusoidal function. At $t = 0$, the bumblebee is at its lowest point above the ground.

a) What does the amplitude of the sinusoidal function represent in this situation?

the radius of the bee's movement, how far out it goes in any direction

3

b) What does the period of the sinusoidal function represent in this situation?

One circular motion of the bee

2

c) What does the equation of the axis of the sinusoidal function represent in this situation? how high the centre of the circle is from the ground

The ground

1

d) If a reflection in the horizontal axis was applied to the sinusoidal function, was the sine function or the cosine function used? Explain

Cosine - the bee starts at a minimum, not the middle

8

6.

Part D – Thinking & Inquiry (12)

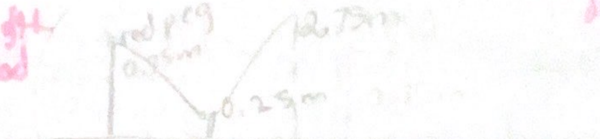
(5)

1. A contestant on a game show spins a wheel that is located on a plane perpendicular to the floor. He grabs the only red peg on the **circumference** of the wheel, which is 0.75m above the floor, and pushes it downward. The red peg reaches a minimum height of 0.25m above the floor and a maximum height of 2.75m above the floor. Find the equation of the sine function that represents the height of the red peg above the floor, as a function of the total distance red peg has traveled.

② p value travelled ~~a quarter~~ eight of the period

$\frac{2.5\pi}{8} = \frac{5\pi}{16}$

starting downward



$y = 1.25\sin(0.8x) + 1.5$

$y = -1.25\sin\left(\frac{4}{5}\left(t + \frac{5\pi}{16}\right)\right) + 1.5$

max = 2.75
min = 0.25

$y = \sin x$
a = 1.25
eoa = 1.5

$eoa = \frac{2.75 + 0.25}{2} = 1.5$

circumference $= 2\pi r = 2\pi(1.25)$

Period 7.8539

Period = one revolution

① p = one revolution (circumference) = amp. Rod.
$= 2\pi r$
$= 2\pi(1.25)$
$= 2.5\pi$

$amp. = \frac{2.75 - 0.25}{2} = 1.25$

$K = \frac{2\pi}{P} = \frac{2\pi}{7.8539} = 0.80$

2. The number of hours of daylight in Australia can be modeled by a trig. function of time, in days, from the beginning of the year (364 days). The longest day of the year is Dec 2 (day 336) with 16.2 hours of daylight. The shortest is June 3 (day 154) with 11.4 hours.

a) Find an equation for this situation.

(5)

(16.2, 336)
(11.4, 154)

max = 336
min = 154

$y = 91\cos(0.017x) + 245$

$y = -2.4\cos\left(\frac{\pi}{182}(d - 154)\right) + 13.8$

use y-values as max/min
max = 16.2
min = 11.4

$eoa = \frac{336 + 154}{2} = 245$

① $P = \frac{2\pi}{k}$
$364 = \frac{2\pi}{k}$
$k = \frac{2\pi}{364} = \frac{\pi}{182}$

$amp. = \frac{336 - 154}{2} = 91$

period: from max to min is 182 days. $182 \times 2 = 364$

$K = \frac{2\pi}{364} = 0.017$

Starts higher than the middle → use cos

② max at t = 336, x should be 0, so it's been moved 336 right

b) Use your answer in a) to find the hours of daylight on Jan. 24 (one decimal).

(2)

$f(24) = 91\cos(0.017(24)) + 245$

$= 32.8$ hours of daylight

5

12